

GSFLOW—A BASIN-SCALE MODEL FOR COUPLED SIMULATION OF GROUND-WATER AND SURFACE-WATER FLOW—PART B. CONCEPTS FOR MODELING SATURATED AND UNSATURATED SUBSURFACE FLOW WITH THE U.S. GEOLOGICAL SURVEY MODULAR GROUND-WATER MODEL

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Abstract

MODFLOW is a ground-water flow model that can be used to assess ground-water resource problems and to make management decisions on the development of ground-water supplies at a variety of scales. It also has the ability to assess the effects of ground-water withdrawals on surface water using several options that simulate ground-water interactions with streams and lakes. The surface processes in MODFLOW do not include surface-water and energy budgets for partitioning precipitation into evapotranspiration, runoff, interflow, and unsaturated flow beneath the soil zone. Typically, runoff, infiltration, and unsaturated flow reaching the water table (assumed ground-water recharge) are estimated externally to MODFLOW. MODFLOW was coupled to the U.S. Geological Survey (USGS) Precipitation-Runoff Modeling System (PRMS) to improve how recharge is estimated for ground-water modeling studies and to provide a complete accounting of the water budget in a watershed or basin. This coupled model is called GSFLOW. Additional code was added to GSFLOW to facilitate the coupling of the two models. The connection in MODFLOW is through a new Unsaturated Zone Flow (UZF1) Package that is designed to route flow from the soil zone used in PRMS to the water table.

INTRODUCTION

The need for coupling MODFLOW to a precipitation-runoff model has increased as a result of concerns regarding the effects of land-use and climate change on the distribution of ground-water recharge. Because the conditions of flow and storage of water both above and below land surface affect recharge, these conditions often need to be simulated together (coupled) to predict changes in water resources. Presently, there are few codes available for modeling large-scale surface-water/ground-water interactions with capabilities of simulating time and space variable precipitation, evapotranspiration, overland flow and interflow, soil-zone storage, and unsaturated flow beneath the soil zone. For this reason, the Modular Ground-Water Model (MODFLOW; McDonald and Harbaugh, 1988; and Harbaugh et al., 2000) was coupled to the Precipitation-Runoff Modeling System (PRMS) (Leavesley et al., 1983) for simulating these processes. The coupled model is named GSFLOW. GSFLOW is designed to conserve mass throughout the model domain. An overall water budget is printed along with optional water budgets for each of the main components.

A new module for PRMS and a new package for MODFLOW were developed for GSFLOW to facilitate the coupling between PRMS and MODFLOW. The original soil-zone module in PRMS was rewritten to allow for coupling to a new package in MODFLOW named the Unsaturated-Zone Flow (UZF1) package. Modification to the soil-zone module in PRMS is discussed in a companion paper (Markstrom et al., 2006) along with the use of GIS spatial data in developing the necessary datasets for GSFLOW. The soil zone is a relatively shallow zone that typically includes the rooting depth of plants. Water stored in the soil zone within PRMS is partitioned into evapotranspiration, interflow, and downward flow beneath the soil zone. Downward flow from the soil zone into the deeper unsaturated zone (UZF1) is dependent on storage in the soil zone and the vertical hydraulic conductivity of the deeper unsaturated zone.

This paper discusses the development of UZF1 (Niswonger et al., in press), which simulates unsaturated flow and storage beneath the soil zone and head-dependent flow to the soil zone when the water table is at or above the bottom of the soil zone. Additionally, adaptations to the MODFLOW Streamflow-Routing (SFR1) Package (Prudic et al., 2004) are described for simulating distributed flow in channels and unsaturated flow beneath channels.

Ground-water flow concepts are discussed with regards to an application of GSFLOW to a 27 km² basin in the Sierra Nevada.

UNSATURATED FLOW BENEATH THE SOIL ZONE

Approach

Several approaches have been used to simulate unsaturated flow for the prediction of recharge in ground-water flow models. A popular approach is to solve Richards' equation using finite-difference or finite-element methods to simulate three-dimensional (3-D) variably saturated flow (Panday and Huyakorn, 2000; Thoms et al., in press). Richards' equation is accurate for simulating unsaturated flow and evapotranspiration; however, Richards' equation is highly non-linear and can be impractical for large problems.

A simpler approach has been used to model unsaturated flow with a one-dimensional (1-D) form of Richards' equation that is solved by the finite-difference technique (Pikul et al., 1974; Refsgaard and Storm, 1995). The 1-D Richards' equation is used to simulate vertical flow through the unsaturated zone, and flow across the water table from the unsaturated zone is applied as recharge to the 3-D ground-water flow equation. This approach significantly reduces the number of computations required to simulate unsaturated flow for large problems. However, this coupling is not straightforward because Richards' equation requires a much shorter length of time and space over which to calculate a solution as compared to the ground-water flow equation. Furthermore, inconsistencies arise among the two equations for the prediction of the water-table elevation.

Unsaturated flow in GSFLOW is simulated as vertical flow similar to the models of Pikul et al. (1974) and Refsgaard and Storm (1995); however, Richards' equation is simplified to resemble a kinematic-wave equation by ignoring the diffusive term (Colbeck, 1972; Smith, 1983). This allows Richards' equation to be solved using the method of characteristics. Because the method of characteristics solution of the simplified Richards' equation does not require vertical discretization of the unsaturated zone, the aforementioned problems associated with coupling Richards' equation to the ground-water flow equation are avoided. This approach for simulating unsaturated flow was implemented in UZF1.

UZF1 simulates 1-D vertical unsaturated flow, evapotranspiration, and ground-water discharge to the soil zone in MODFLOW. Thus, it handles all fluxes of water between MODFLOW and the soil-zone module in PRMS. The kinematic approximation of Richards' equation can be written to consider evapotranspiration losses as (Colbeck, 1972; Smith, 1983; Charbeneau, 1984):

$$\frac{\partial \theta}{\partial t} + \frac{\partial K(\theta)}{\partial z} + i = 0. \quad (1)$$

where θ is the volumetric water content (volume of water per volume of rock); z is the elevation in the vertical direction (length); $K(\theta)$ is the unsaturated hydraulic conductivity as a function of water content (length per time); i is the ET rate per unit depth (length per time per length); and t is time. Application of the method of characteristics to equation 1 results in the following set of coupled ordinary differential equations (Niswonger et al., in press):

$$\frac{dz}{dt} = \frac{\partial K(\theta)}{\partial \theta} = v(\theta), \quad (2a)$$

$$\frac{d\theta}{dt} = -i, \quad (2b)$$

$$\frac{d\theta}{dz} = \frac{-i}{v(\theta)}. \quad (2c)$$

Where $v(\theta)$ is the characteristic velocity restricted to the downward (positive z) direction (length per time). Equation 2a provides the velocity of waves that represent wetting and drying in the unsaturated zone. Equation 2b provides the change in water content at the front of a wave through time, and equation 2c provides the change in water content along a wave profile (behind the front). Equations 2a, b, and c are separable, and can be integrated to find analytical expressions that are used by UZF1 to simulate vertical unsaturated flow.

Coupling UZF1 to MODFLOW

Recharge to the water table in an unconfined aquifer is subtracted from the right-hand side of the system of equations that are solved by MODFLOW:

$$AX = B - Q_{UZF1} , \quad (3)$$

where A is a matrix containing the coefficients of the conductance equations (HCOF array) that are solved by MODFLOW; X is a one-dimensional vector containing the ground-water heads that are solved by MODFLOW; B is a one-dimensional vector containing all known terms in the conductance equations that are not multiplied by unknown head values (RHS array); and Q_{UZF1} is the volumetric rate (volume per time) of recharge computed in a given model cell from UZF1.

Recharge simulated using UZF1 is dependent on the location of the water table and how it varies with time. This is because MODFLOW does not account for storage in the unsaturated zone. Rather, MODFLOW uses specific yield to estimate changes in ground-water storage. Thus when the water table increases, storage in the unsaturated zone over which the rise occurred is added to recharge. When the water table declines, the thickness of the unsaturated zone is increased and a wetting front must advance through the interval of the decline before there is recharge.

Thus, UZF1 is coupled to MODFLOW within the nonlinear iteration loop. However, because ground-water recharge is dependent on the amount of storage and the downward flux rate in the unsaturated zone, there is no general equation that defines the relation between recharge and ground-water head, such as the conductance equation in the Stream (SFR1) Package (Prudic et al., 2004). Thus, UZF1 does not affect the HCOF array.

The UZF1 Package also allows for ground-water seepage into the soil zone whenever the water table in a cell is higher than the elevation of the bottom of the soil zone. The volumetric rate of ground-water seepage to the soil zone is calculated on the basis of the following equation:

$$Q_{gw} = A_{cell} K_v (h - celtop) / (0.5celthk) , \quad (4)$$

where h is the water-table elevation; $celtop$ is the elevation of the bottom of the soil zone; A_{cell} is the map area of the model cell (equal to the column length times the row length of the model grid); K_v is the vertical hydraulic conductivity of the model cell; and $celthk$ is the thickness of the model cell. The value that is subtracted from the HCOF array in MODFLOW (corresponding to the A matrix in equation 3) is equal to $A_{cell} K_v / (0.5celthk)$. The value subtracted from the RHS array in MODFLOW (corresponds to the B matrix in equation 3) is equal to $A_{cell} K_v celtop / (0.5celthk)$.

STREAMFLOW

Distributed Streamflow Routing in Channels

Similar to the approach used to simulate unsaturated flow, a kinematic wave approximation of the Saint-Venant equations was used to simulate streamflow (Lighthill and Whitham, 1955). However, unlike UZF1, the adapted version of Stream Routing Package, named SFR2 (Niswonger and Prudic, 2006), solves the kinematic-wave equation using the implicit finite-difference technique. The kinematic-wave equation for routing flow in streams can be written:

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q , \quad (5)$$

where Q is discharge (length cubed per time); A is the flow cross-sectional area (length squared); q is the lateral inflow per unit length (length squared per time); and x (length) and t (time) are space and time coordinates. SFR2 considers momentum associated with the bed slope and channel friction based on Manning's equation, specified rating curves, or general power law relationships as described by Leopold et al. (1992). Momentum due to the water-surface slope and acceleration terms in the Saint-Venant equations are neglected in the kinematic-wave approximation. SFR2 can route flow in non-prismatic channels.

Coupling SFR2 to MODFLOW

Stream reaches are coupled to MODFLOW cells based on a Darcy-type equation for flow through a streambed, which can be written as:

$$Q_L = \frac{KwL}{m}(h_s - h_a), \quad (6)$$

where Q_L is a volumetric flow between a given section of stream and volume of aquifer (volume per time); K is the hydraulic conductivity of streambed sediments (length per time); w is a representative width of stream (length); L is the length of stream corresponding to a volume of aquifer (length); m is the thickness of streambed sediments extending from the top to the bottom of the streambed (length); h_s is the head in the stream determined by adding stream depth to the elevation of the streambed (length); and h_a is the head in the aquifer beneath the streambed (length).

The same method used to simulate unsaturated flow in UZF1 is used to simulate vertical flow beneath the channel whenever the head in the aquifer is below the bottom of the streambed (Niswonger and Prudic, 2006). Nonlinearities associated with the relation among stream depth, width, aquifer head, and flow between the stream and aquifer are solved inside of the MODFLOW nonlinear iteration loop.

SAGEHEN CREEK APPLICATION

GSFLOW was used to model surface and subsurface flow in the Sagehen basin, which is a USGS Hydrologic Benchmark Network Basin located on the east slope of the northern Sierra Nevada near Truckee, California (Figure 4). The basin drains an area of 27 km² and ranges in altitude from 1,926 to 2,663 m above mean sea level. The Sagehen basin consists of granitic rocks overlain by volcanic rocks and a thin veneer of alluvium. The aerially averaged annual precipitation is about 970 mm and the annual hydrograph is dominated by snowmelt. Daily discharge records are available beginning October 1953.

The simulation consisted of a single model layer with 73 rows and 81 columns in which all cells had a constant width and length equal to 90 m. The top elevations of model cells were set equal to estimated altitudes of the soil-zone bottom and the bottom elevations of the model cells were assigned altitudes that ranged from 100 m below the soil zone in the valleys to as much as 200 m below the soil zone on the ridges. Bottom elevations of the model ranged from 1,830 to 2,480 m above mean sea level. The results presented in this paper are based on a simulation period that was one-year long beginning on March 7, 1983 and ending March 7, 1984. The simulation period was divided into 365 one-day-long stress periods. Initial heads and unsaturated-zone water contents were taken from the results of the steady-state MODFLOW simulation. One time step was simulated for each stress period. No flow was allowed along the bottom and sides of the model except three cells beneath and adjacent to the stream at the basin outlet were specified as constant-head cells.

Model Calibration

The Layer-Property Flow (LPF) Package in MODFLOW (Harbaugh et al., 2000) was used and both horizontal and vertical hydraulic conductivity (K) values were specified. Because only one layer was specified in the model, the vertical K of the aquifer only affected ground-water seepage to the soil zone and flow in the unsaturated zone. All cells were specified as convertible (unconfined). The horizontal K ranged from 0.004 m/d on the ridges to 0.24 m/d in the valleys for the initial simulation (Figure 1). A lower K was specified on the ridges because the volcanic rocks are near the surface, whereas the volcanic rocks in the valleys are covered with alluvium. The K within each cell was assumed isotropic. Specific storage was set to $5 \times 10^{-7} \text{ m}^{-1}$, and the specific yield was specified as 0.05 on the ridges and 0.25 in the valleys near streams.

The development of the GSFLOW model for the Sagehen basin required calibration of a PRMS model and a MODFLOW model. The calibration process was carried out by first calibrating the models independently; PRMS was calibrated for a ten-year period and MODFLOW was calibrated as steady-state. The companion paper in this issue (Markstrom et al., 2006) briefly describes the calibration procedure used for calibrating PRMS. Only the calibration of the MODFLOW model is described in this paper.

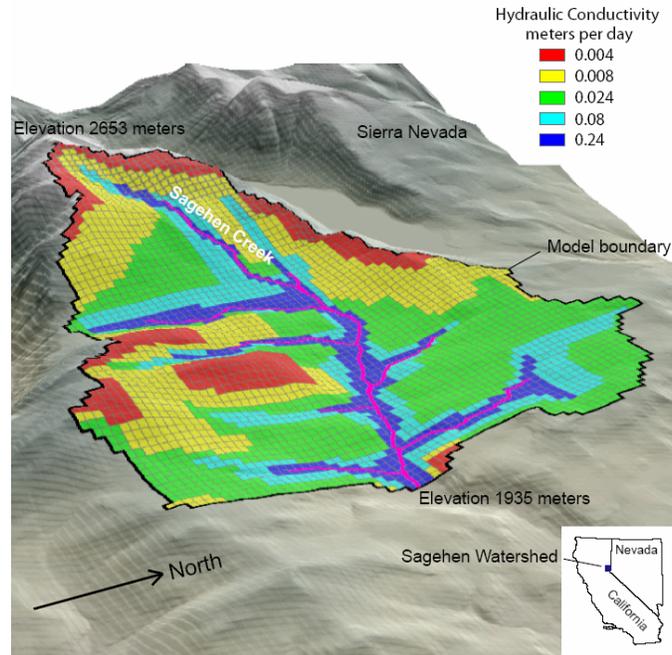


Figure 1 Modeled area of the Sagehen basin, spatial discretization, and hydraulic conductivity (K) distribution.

The distribution of K used for UZF1 and MODFLOW was created initially on the basis of the surface geology and was adjusted during calibration. The steady-state simulation assumed that the spatially varying ground-water recharge was proportional to the distribution of mean annual precipitation. Precipitation was distributed according to 860 mm of precipitation below an altitude of 2,100 m and 1,140 mm of precipitation above an altitude of 2,100 m. The range in ground-water recharge for the steady-state simulation was determined by approximating the mean daily discharge at the outlet of the Sagehen basin in early December. Refinement to the independent model calibrations was made during GSFLOW simulations to variables that control the exchange of water between PRMS and MODFLOW. These variables are the coefficients on the rate equations that control flow between the soil zone and the underlying unsaturated zone or between the soil zone and ground water when the water table is above the elevation of the bottom of the soil zone.

The steady-state simulation resulted in a calculated water table that was as much as 80 m below land surface along the ridges and at or slightly above land surface in the lowest parts of the valleys next to streams. Although no observation wells have been drilled on the ridges in the basin, the maximum depth to ground water was based on depths measured in wells near ridges elsewhere in the northern Sierra Nevada.

Sensitivity of Horizontal and Vertical Hydraulic Conductivity

Only those results associated with subsurface flow beneath the soil zone are presented herein. The companion paper in this issue (Markstrom et al., 2006) includes comparisons between the measured and simulated streamflow. A sensitivity analysis was done to analyze the effects of varying horizontal and vertical hydraulic conductivity beneath the soil zone on infiltration, recharge, and ground-water discharge to the soil zone. Ground-water discharge to the soil zone includes infiltration from the soil zone but is rejected because there no unsaturated zone. Results of ground-water discharge to the soil zone are presented in this paper because this discharge is part of the ground-water budget and is generated as a result of the distribution of hydraulic conductivity in the subsurface below the soil zone.

The vertical and horizontal hydraulic conductivity (K) beneath the soil zone were increased uniformly by one order of magnitude to create a medium-K distribution, and by two orders of magnitude to create a high-K distribution. The hydraulic conductivity was not decreased below the initial values because it was determined that results were much less sensitive to decreases in K than to increases in K. Steady-state simulations were run for each K distribution to establish initial conditions that corresponded to each specific K distribution.

Total infiltration rates into the unsaturated zone increased proportionately to the increase in K during times when total water content within the soil zone was high due to spring snowmelt. The quantity of water available to infiltrate into the unsaturated zone beneath the soil zone during spring snowmelt was limited by the value of the vertical hydraulic conductivity in the unsaturated zone (Figure 2). However, total infiltration rates into the unsaturated zone were less sensitive to the value of the vertical hydraulic conductivity following the spring snowmelt, because infiltration was limited by the available water in the soil zone (Figure 2).

Ground-water recharge rates did not increase proportionately to the increase in hydraulic conductivity. The medium-K distribution resulted in greatest total recharge rates during the snowmelt period (Figure 3). Much of this recharge occurred where the water table was near the bottom of the soil zone. Maximum recharge rates were similar for the high- and low-K distributions. These unexpected results are attributed to the thickness of the unsaturated zone on the timing and rate of recharge, as well as to changes in lateral flow caused by increasing the horizontal hydraulic conductivity.

The higher recharge rates for the medium-K distribution compared with the low-K distribution are because of greater vertical flow through a similar thickness of the unsaturated zone. The lower recharge rates for the high-K distribution compared with the medium-K distribution was caused by a much lower water table, which produced a greater thickness of the unsaturated zone that delayed recharge. This is similar in manner to the slow drainage through a less thick unsaturated zone for the low-K distribution. Additionally, the high-K distribution allowed for more water to be stored in the unsaturated zone, which recharged ground water much longer after the spring snowmelt than either the low- and medium-K distributions. Ground-water levels were the deepest in the upland areas for the high-K distribution because ground water flowed more readily to the streams. The upland areas comprise a larger proportion of the total basin area compared with the valley lowland areas; consequently, a change in the K values in these areas can greatly change the timing and magnitude of recharge in the basin.

Ground-water discharge to the soil zone also did not increase proportionately to the increase in hydraulic conductivity, and was similar in character as the recharge rates (Figure 4). Ground-water discharge was highest for the medium-K distribution only during the period of high infiltration, because the water table was above the bottom of the soil zone over a large area of the basin. Consequently, there was little delay between infiltration, recharge, and ground-water discharge in those areas. Ground-water discharge to the soil zone was highest for the high-K distribution once snowmelt ceased and the soil zone drained. Ground-water discharge following snowmelt was limited to a few areas, primarily near the valley bottoms, for all three K distributions. Thus, the areas contributing to ground-water discharge following snowmelt were similar for all three K distributions, although the quantity of ground-water discharge increased proportionately to the increase in hydraulic conductivity.

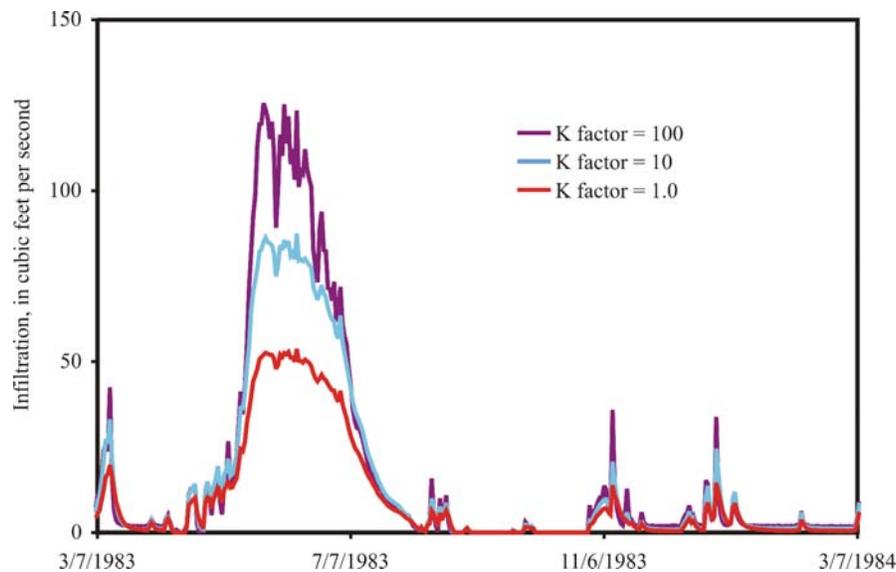


Figure 2 Effects of subsurface hydraulic conductivity on total infiltration into the unsaturated zone beneath the soil zone.

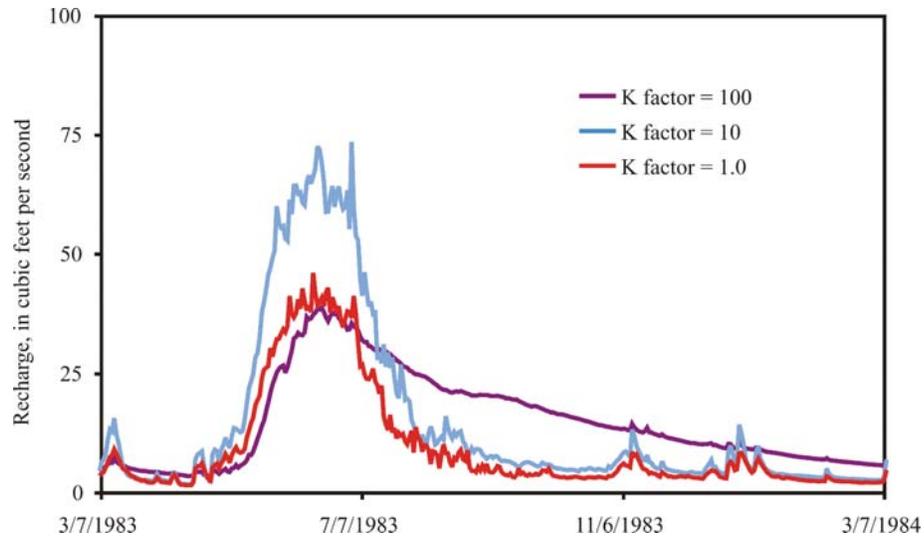


Figure 3 Effects of subsurface hydraulic conductivity on total recharge.

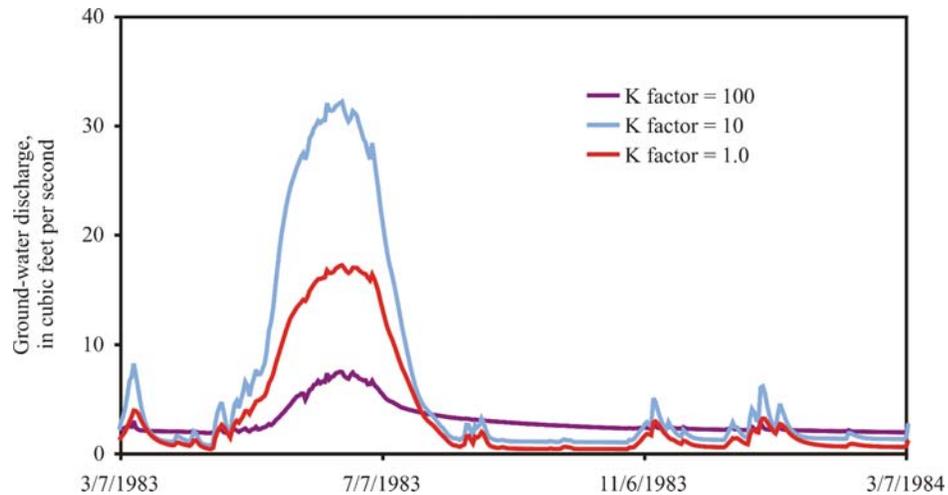


Figure 4 Effects of subsurface hydraulic conductivity on total basin ground-water discharge to the soil zone.

DISCUSSION AND CONCLUSIONS

GSFLOW is a coupled precipitation-runoff and ground-water flow model that provides mass balances and exchange rates among different hydrologic zones, including land surface, soil zone, unsaturated and saturated subsurface zones. GSFLOW is applicable for simulating coupled precipitation, runoff and ground-water flow over large areas because it relies on an efficient method for simulating unsaturated flow. Coupled precipitation-runoff and ground-water flow models that incorporate 3-D Richards' equation to simulate flow in the subsurface require much smaller grid cells and time steps, which constrains applications of such models to relatively small areas. The trade-offs of a simpler model are that the unsaturated zone must be homogeneous in the vertical direction and capillary gradients are neglected. However, the more rigorous equation may not provide more accuracy when modeling unsaturated flow over large areas because of uncertainty in the many input variables that are required to solve the 3-D Richards' equation.

A sensitivity analysis of the effects of hydraulic conductivity in the subsurface zone below the soil zone on infiltration, recharge, and ground-water discharge to the soil zone showed that the thickness of the unsaturated zone can have a significant effect on the timing and rate of recharge in the Sagehen basin. These results would likely apply to other basins that exhibit significant relief, such that the thickness of the unsaturated zone is sensitive to hydraulic conductivity. The timing and rate of recharge affect changes in ground-water storage, and therefore, water availability for human populations and wildlife.

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