

*Generalized Random Tessellation Stratified  
(GRTS)  
Spatially-Balanced Survey Designs  
for Aquatic Resources*

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  - Barbara Rosenbaum, INDUS Corporation
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- EMAP Surface Waters Research Group
  - Users: States, EPA Regions, & others



# Overview

- Aquatic resource characteristics
- Sample frame
  - GIS coverages
  - Imperfect representation of target population
- GRTS theory
- GRTS implementation



## *Features in Space as GIS objects*

Feature	Points	Lines	Polygons
Lakes	Individual lakes		Lake area
Streams	Segments	Linear network	
Estuaries			Estuarine area
Wetlands	Depressional wetlands		Wetland area
Roads/trails		Linear network	
Hydrologic Units	As points		As areas
Terrestrial Vegetation			As areas



# *Natural Resource Characteristics*

- Types of natural resources
  - Area polygons: large lakes and reservoirs, estuaries, coastal waters, everglades, forests, landscapes
  - Linear networks: streams, rivers, roads, trails
  - Discrete points: small lakes, stream reaches, prairie pothole wetlands, hydrologic units (“watersheds”), buildings
- Target population
  - Finite in a bounded geographic region: collection of points
  - Continuous in a bounded geographic region
    - As linear network
    - As collection of polygonal areas
- Generalizations
  - Geographic region may be 1-dimensional (p-dimensional)
  - “Space” may be defined by other auxiliary variables



# *Typical Aquatic Sample Frames*

- GIS coverages do exist for aquatic resources
- National Hydrography Dataset (NHD)
  - Based on 1:100,000 USGS maps
  - Combination of USGS Digital Line Graph (DLG) data set and USEPA River Reach File Version 3 (RF3)
  - Includes lakes, ponds, streams, rivers
- Sample frames derived from NHD
  - Use GIS to extract frame to match target population
  - Enhance NHD with other attributes used in survey design
- Issues with NHD
  - Known to include features not of interest (over-coverage)
  - Known to exclude some aquatic resources (under-coverage)



## *Other Natural Resource Frames*

- Omernik ecoregions digital maps
- National Wetland Inventory (NWI) digital maps
- Landcover/landuse digital maps



# *Generalized Random Tessellation Stratified (GRTS) Survey Designs*

- Probability sample producing design-based estimators and variance estimators
- Gives another option to simple random sample and systematic sample designs
  - Simple random samples tend to “clump”
  - Systematic samples difficult to implement for aquatic resources and do not have design-based variance estimator
- Emphasize spatial-balance
  - Every replication of the sample exhibits a spatial density pattern that closely mimics the spatial density pattern of the resource



## *GRTS Implementation Steps*

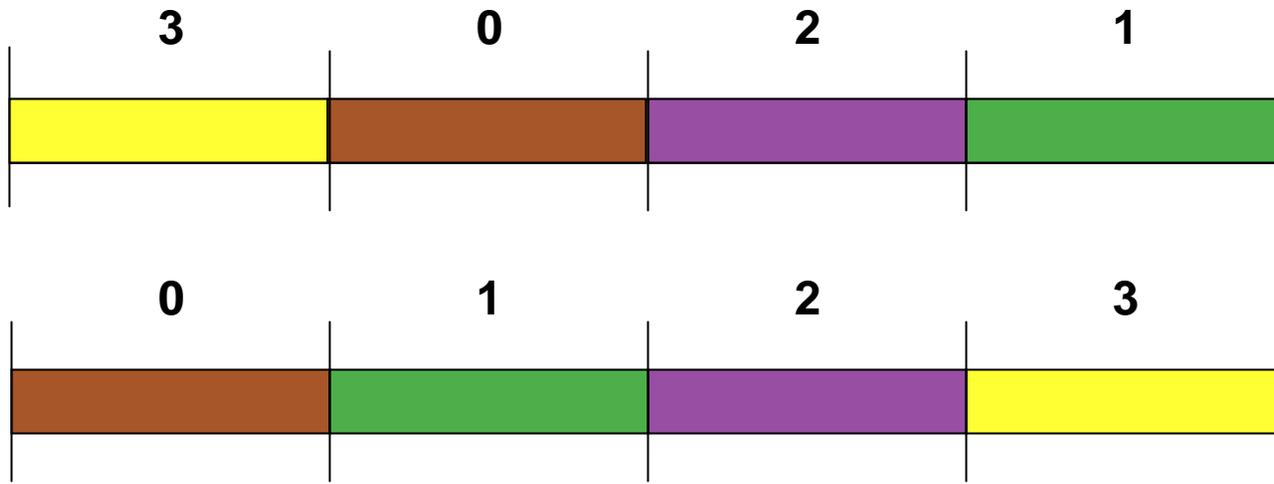
- Concept of selecting a probability sample from a sampling line for the resource
- Create a hierarchical grid with hierarchical addressing
- Randomize hierarchical addresses
- Construct sampling line using randomized hierarchical addresses
- Select a systematic sample with a random start from sampling line
- Place sample in reverse hierarchical address order



# *Ohio River GRTS*

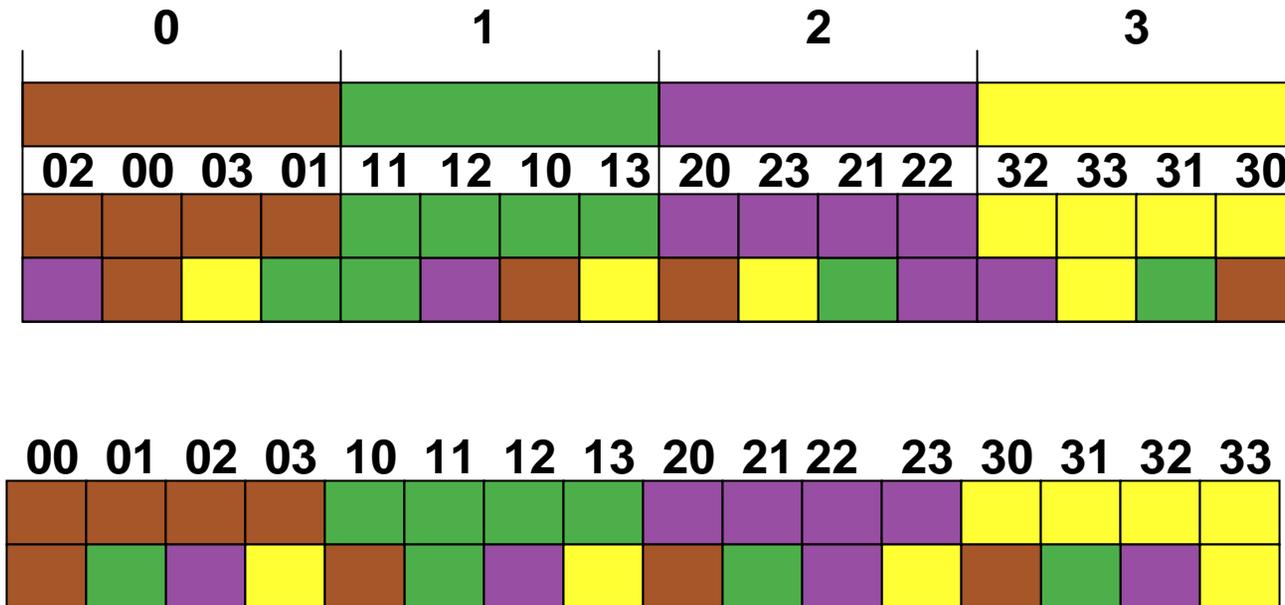


## *Create Straight Line All Reaches*



Create Line for Ohio River (length)  
Divide into 4 segments  
Create Random Sequence (3, 0, 2, 1)  
Assign address & color  
Sort address  
Repeat for each segment

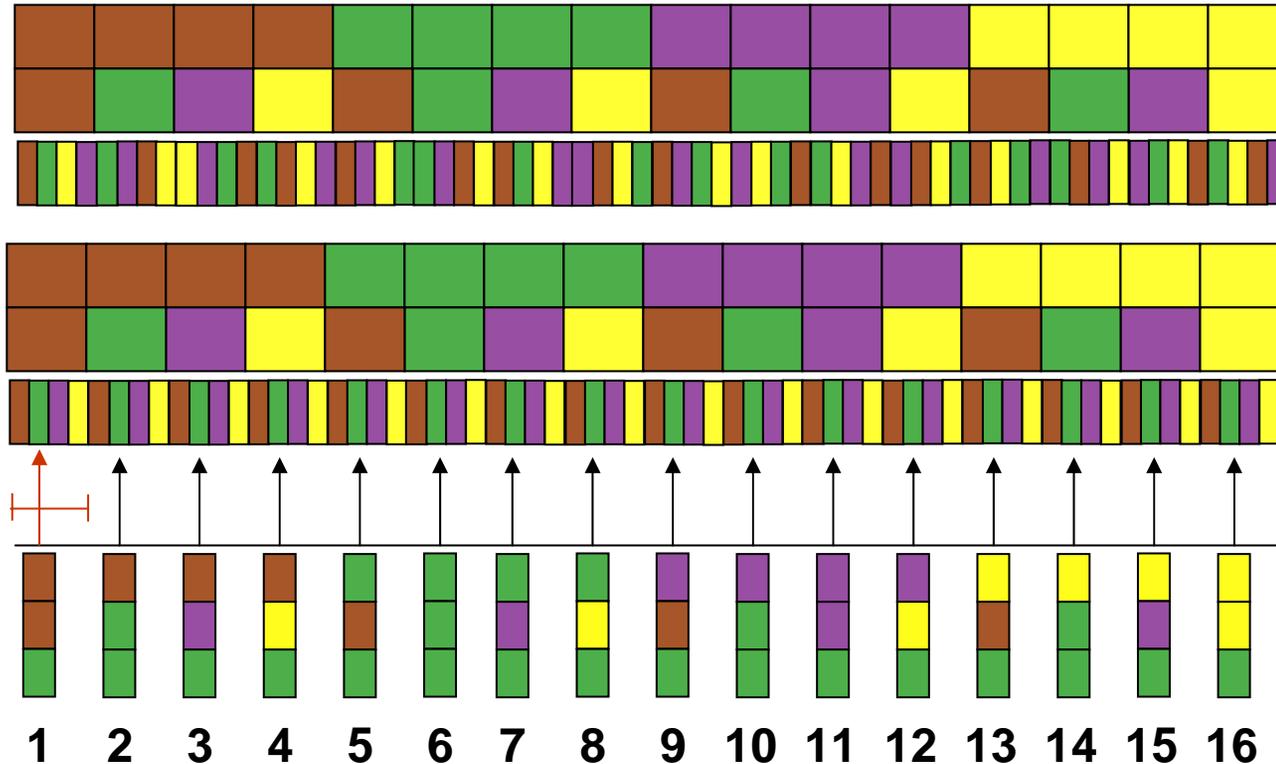
# Repeating Process



Divide each segment into 4 segments  
 Create New Random Sequences  
 (2,0,3,1) (1,2,0,3) (0,3,1,2) (2,3,1,0)  
 Assign address & colors  
 Sort



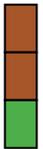
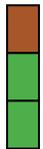
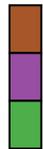
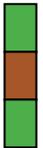
# Selecting 16 Sample Points



Subdivide, Create Random Sequence, Assign Address & Colors  
Sort Addresses  
Random Starting point, Uniformly Sample Line  
Assign Sequence Number to Each Point



# Reverse Hierarchical Order

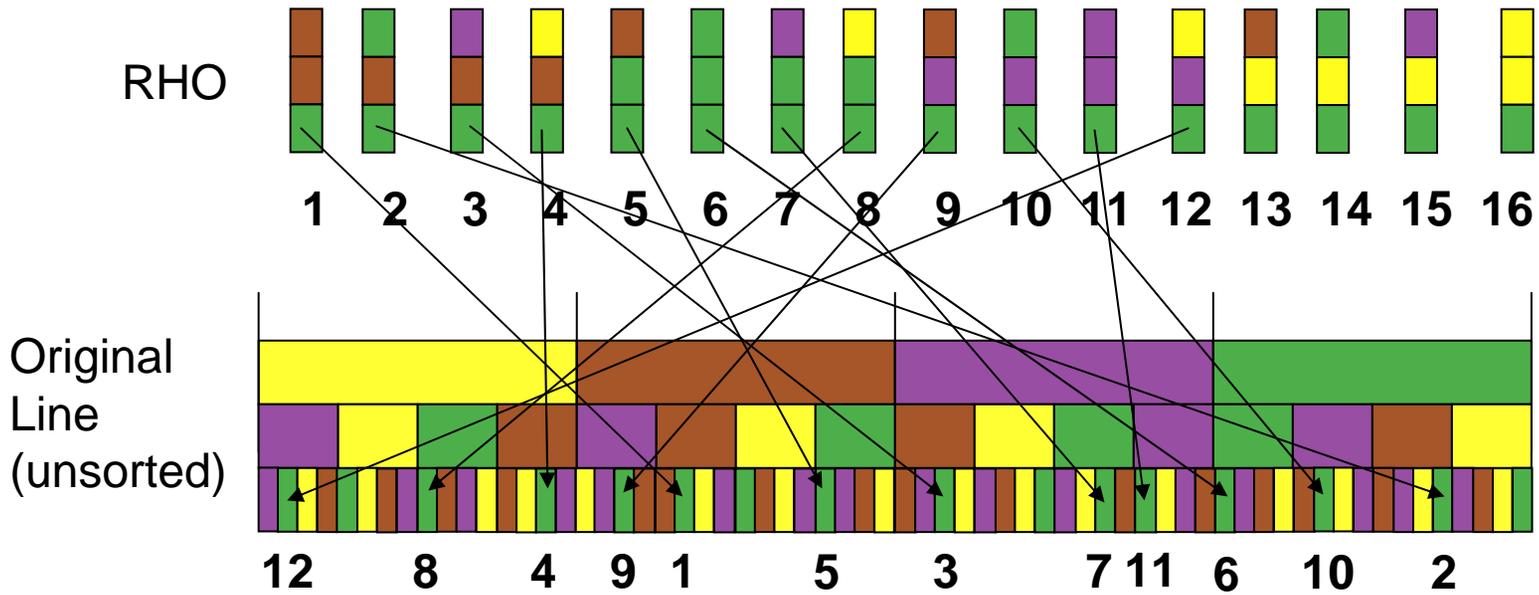
Original Order																
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Base4	00	01	02	03	10	11	12	13	20	21	22	23	30	31	32	33
Reverse Base4	00	10	20	30	01	11	21	31	02	12	22	32	03	13	23	33
Sort	00	01	02	03	10	11	12	13	20	21	22	23	30	31	32	33
RHO Site Number																
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Create Base4 Addresses  
Reverse Address Digits

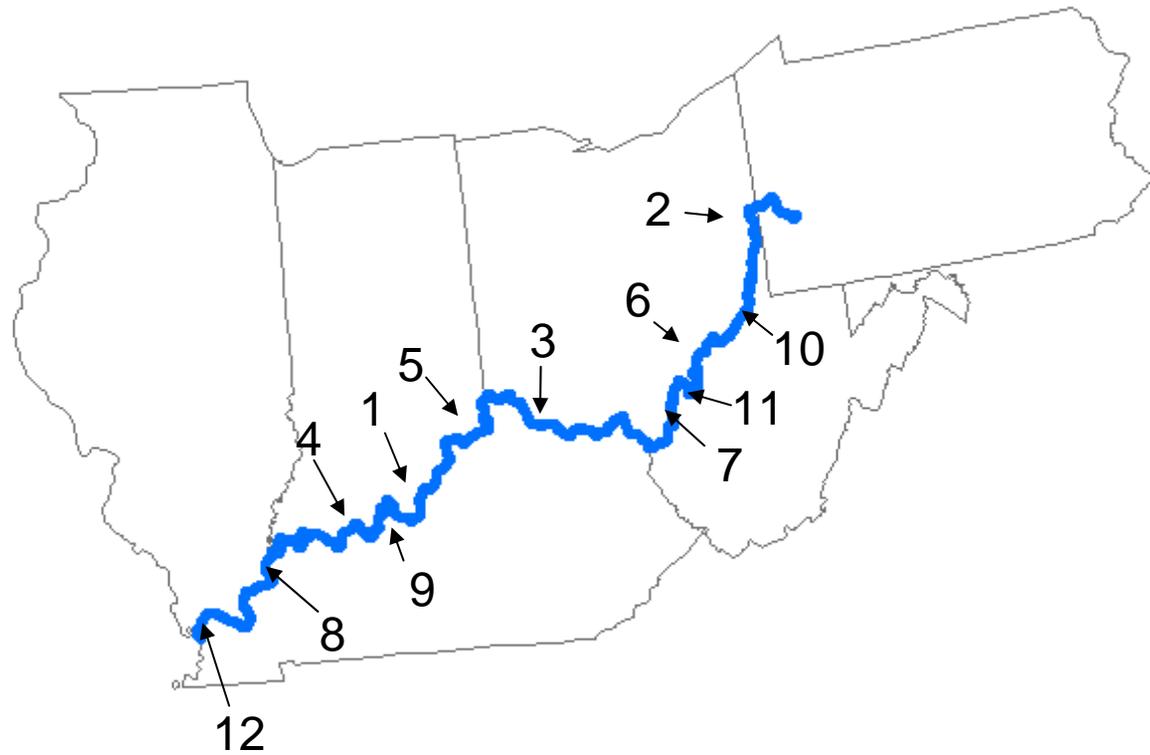
Sort  
Assign RHO Site Nos.



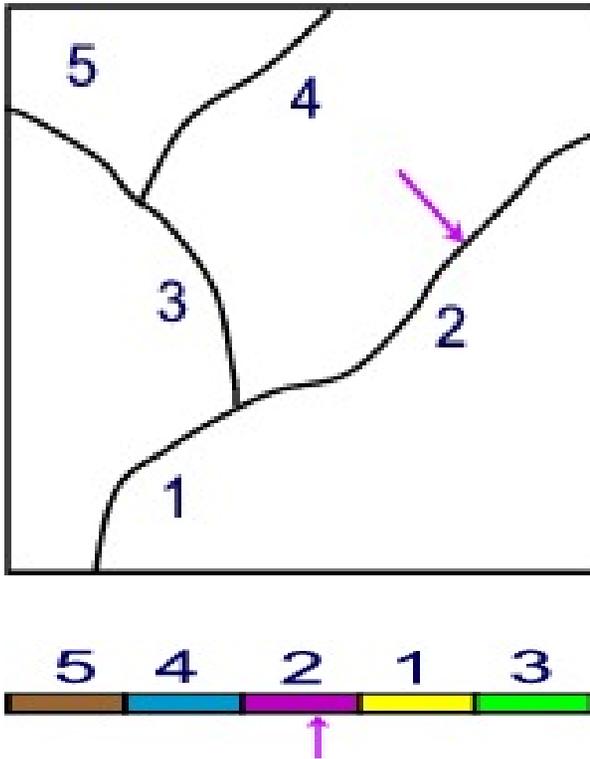
# Map Sites



# *Ohio River Sites*

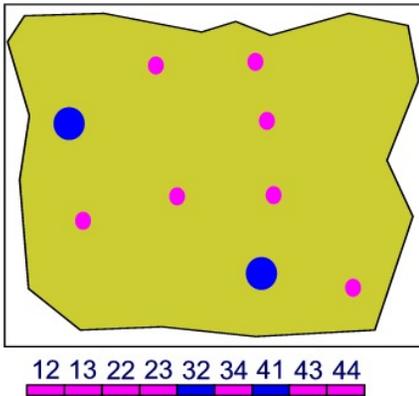


# Selecting a Probability Sample from a Sampling Line: Linear Network Case

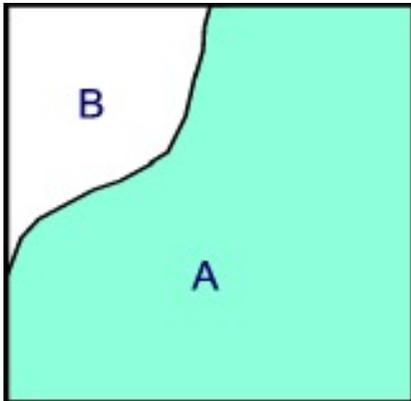


- Place all stream segments in frame on a linear line
  - Preserve segment length
  - Identify segments by ID
- In what order do place segments on line?
  - Randomly
  - Systematically (minimal spanning tree)
  - Randomized hierarchical grid
- Systematic sample with random start
  - $k=L/n$ ,  $L$ =length of line,  $n$ =sample size
  - Random start  $d$  between  $[0,k)$
  - Sample:  $d + (i-1)*k$  for  $i=1, \dots, n$

# Selecting a Probability Sample from a Sampling Line: Point and Area Cases



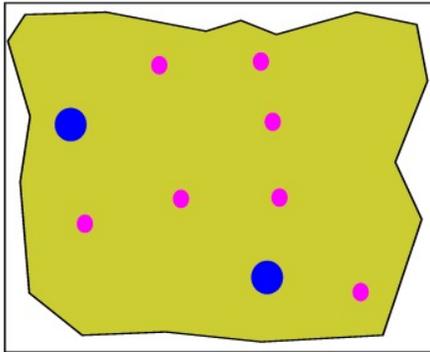
- Point Case:
  - Identify all points in frame
  - Assign each point unit length
  - Place on sample line



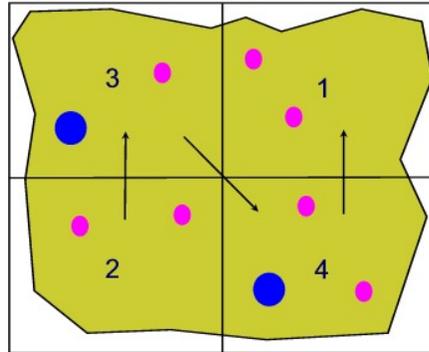
- Area Case:
  - Create grid covering region of interest
  - Generate random points within each grid cell
  - Keep random points within resource (A)
  - Assign each point unit length
  - Place on sample line

# Randomized Hierarchical Grid

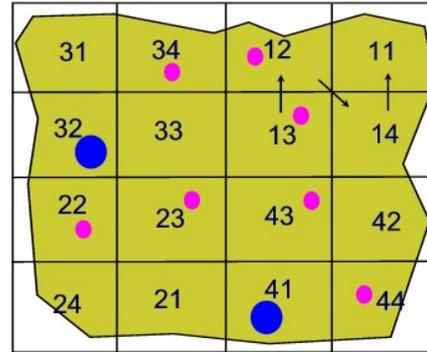
Step 1



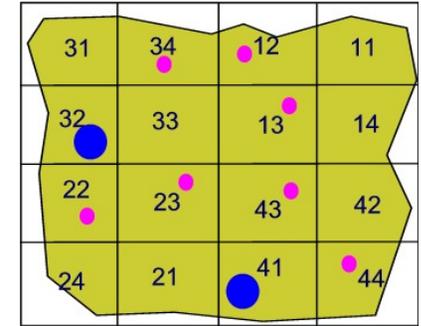
Step 2



Step 3



Step 4

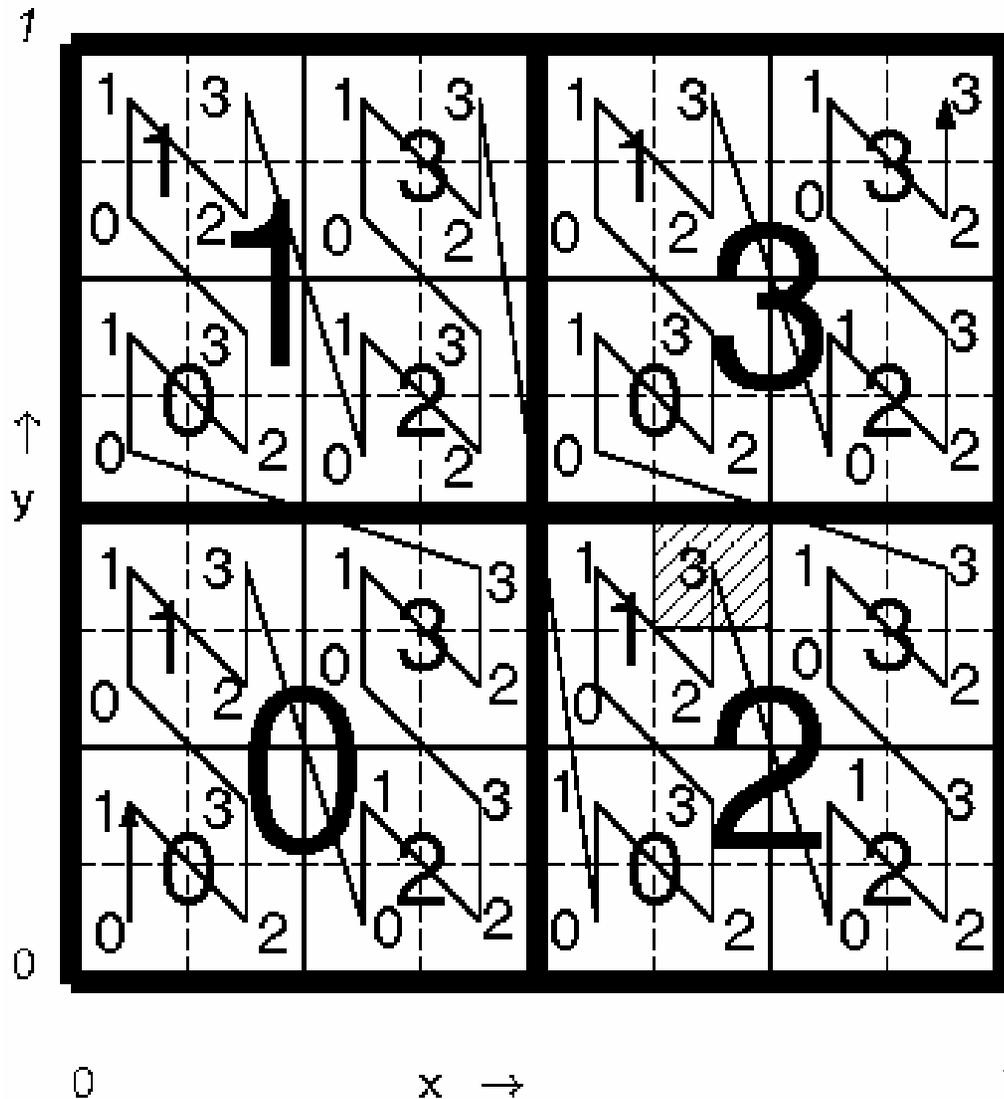


12 13 22 23 32 34 41 43 44

- Step 1: Frame: Large lakes: blue; Small lakes: pink; Randomly place grid over the region
- Step 2: Sub-divide region and randomly assign numbers to sub-regions
- Step 3: Sub-divide sub-regions; randomly assign numbers independently to each new sub-region; create hierarchical address. Continue sub-dividing until only one lake per cell.
- Step 4: Identify each lake with cell address; assign each lake length 1; place lakes on line in numerical cell address order.



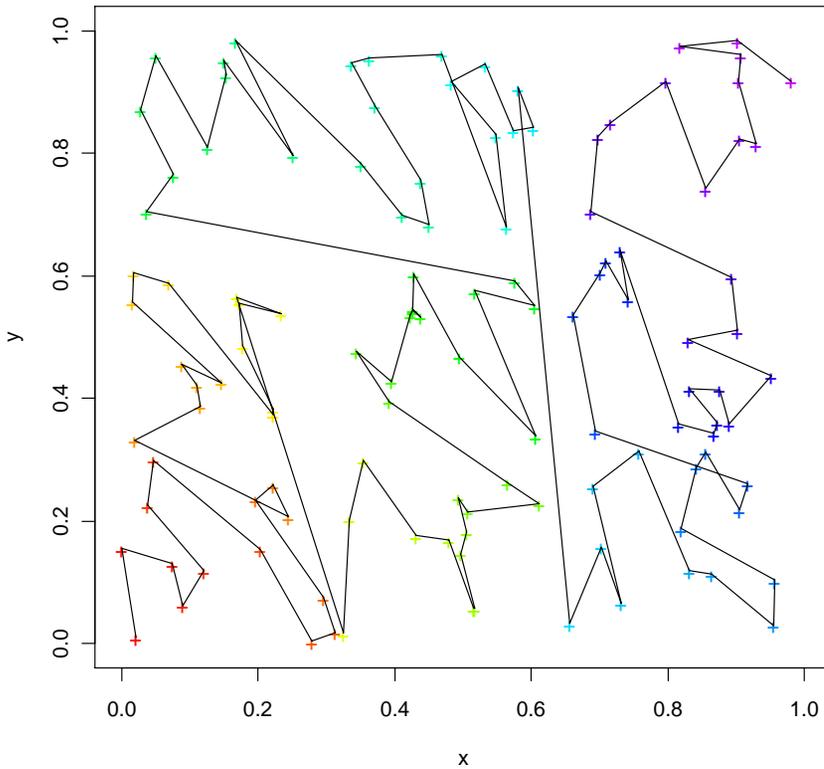
# Hierarchical Grid Addressing



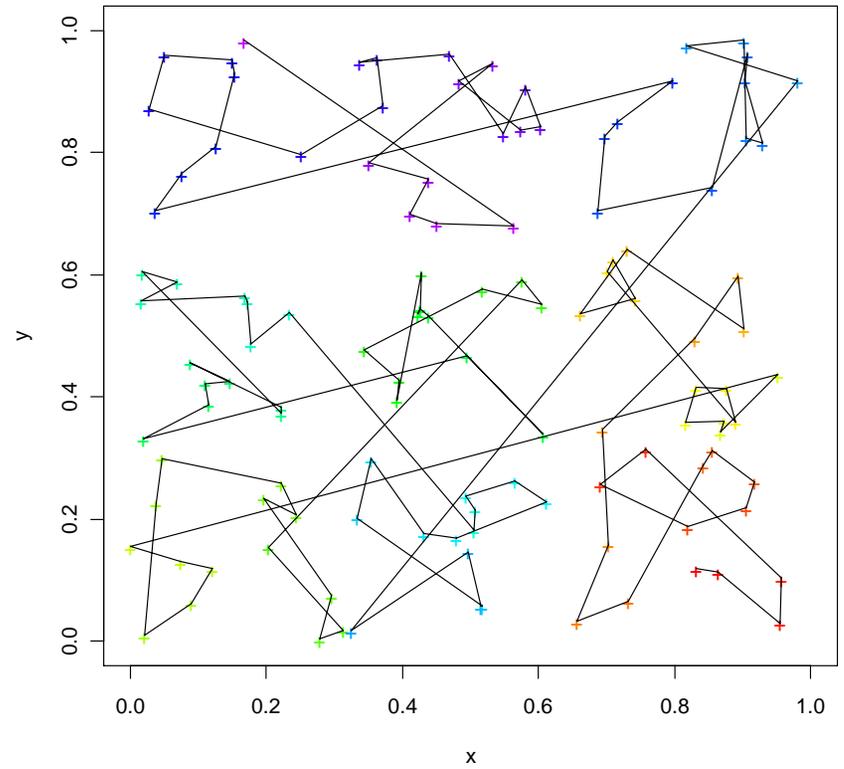
**213: hierarchical address**

# Population of 120 points

Hierarchical Order



Hierarchical Randomized Order



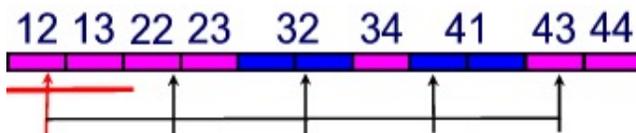
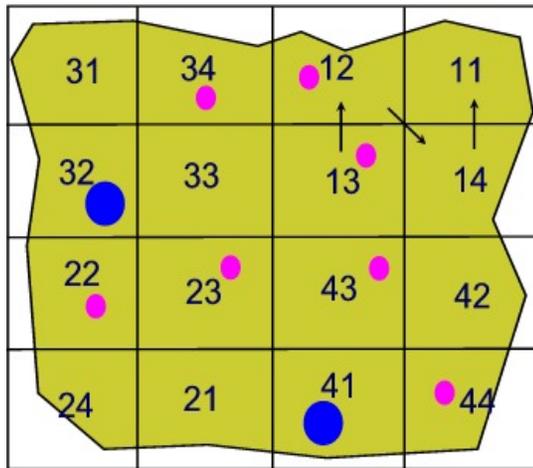
RHO	Reverse Base4	Base4	Original Order
1	00	00	1
2	01	10	5
3	02	20	9
4	03	30	13
5	10	01	2
6	11	11	6
7	12	21	10
8	13	31	14
9	20	02	3
10	21	12	7
11	22	22	11
12	23	32	15
13	30	03	4
14	31	13	8
15	32	23	12
16	33	33	16

## *Reverse Hierarchical Order*

- Construct reverse hierarchical order
  - Order the sites from 1 to n
  - Create base 4 address for numbers
  - Reverse base 4 address
  - Sort by reverse base 4 address
  - Renumber sites in RHO
- Why use reverse hierarchical order?
  - Results in any contiguous set of sample sites being spatially-balanced
  - Consequence: can begin at the beginning of list and continue using sites until have required number of sites sampled in field



# Unequal Probability of Selection



- Assume want large lakes to be twice as likely to be selected as small lakes
- Instead of giving all lakes same unit length, give large lakes twice unit length of small lakes
- To select 5 sites divide line length by 5 ( $11/5$  units); randomly select a starting point within first interval; select 4 additional sites at intervals of  $11/5$  units
- Same process is used for points and areas (using random points in area)

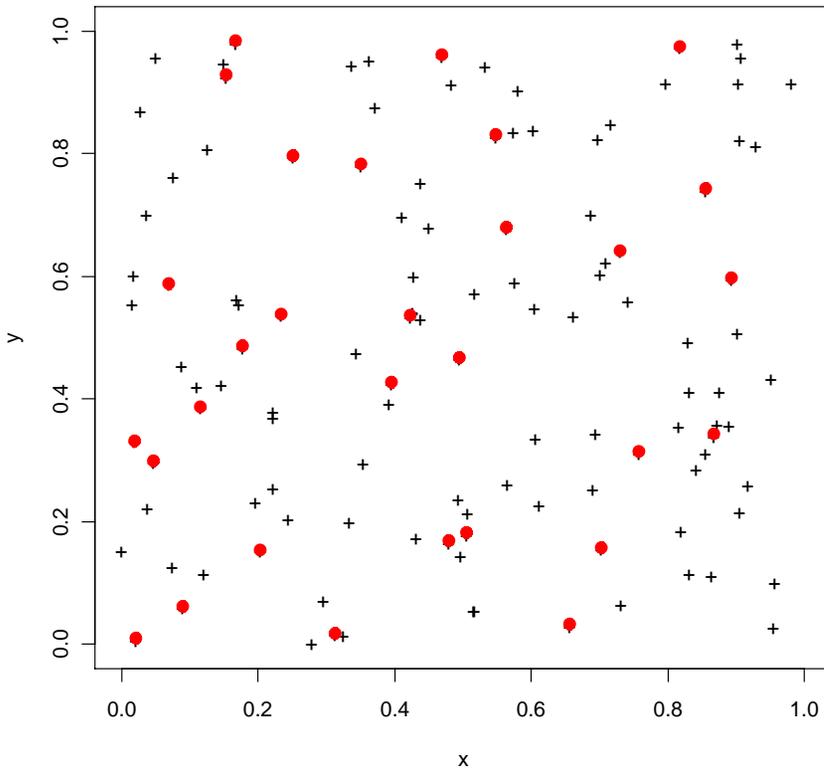
# *Complex Survey Designs based on GRTS*

- Stratified GRTS: apply GRTS to each stratum
- Unequal probability GRTS: adjust unit length based on auxiliary information (eg lake area, strahler order, basin, ecoregion)
- Oversample GRTS:
  - Design calls for  $n$  sites; some expected non-target, landowner denial, etc; select additional sites to guarantee  $n$  field sampled
  - Apply GRTS for sample size  $2n$ ; place sites in RHO; use sites in RHO
- Panels for surveys over time
- Nested subsampling
- Two-stage sampling using GRTS at each stage
  - Example: Select USGS 4<sup>th</sup> field Hucs; then stream sites within Hucs

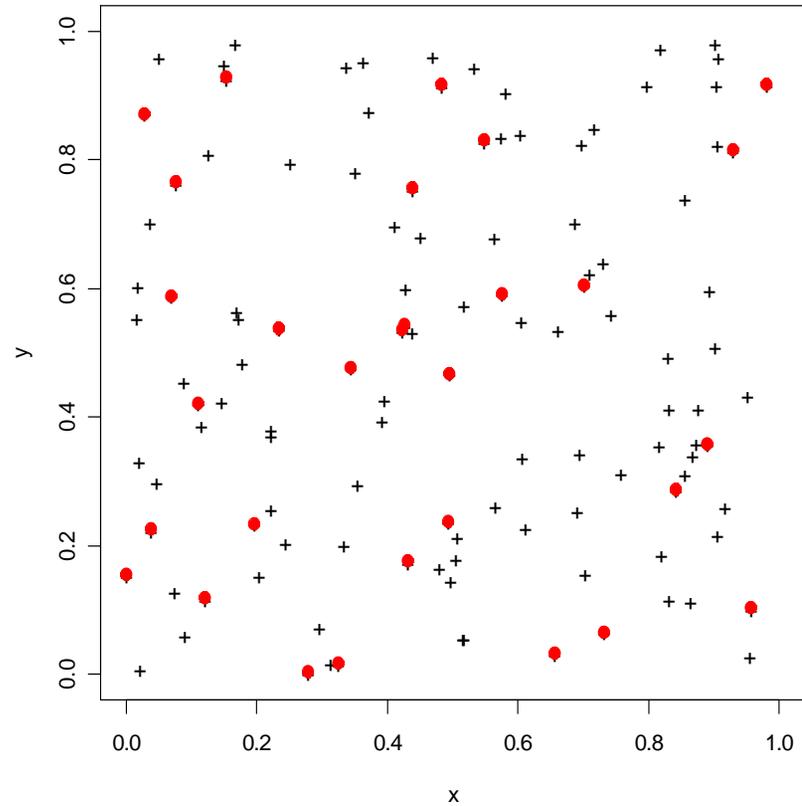


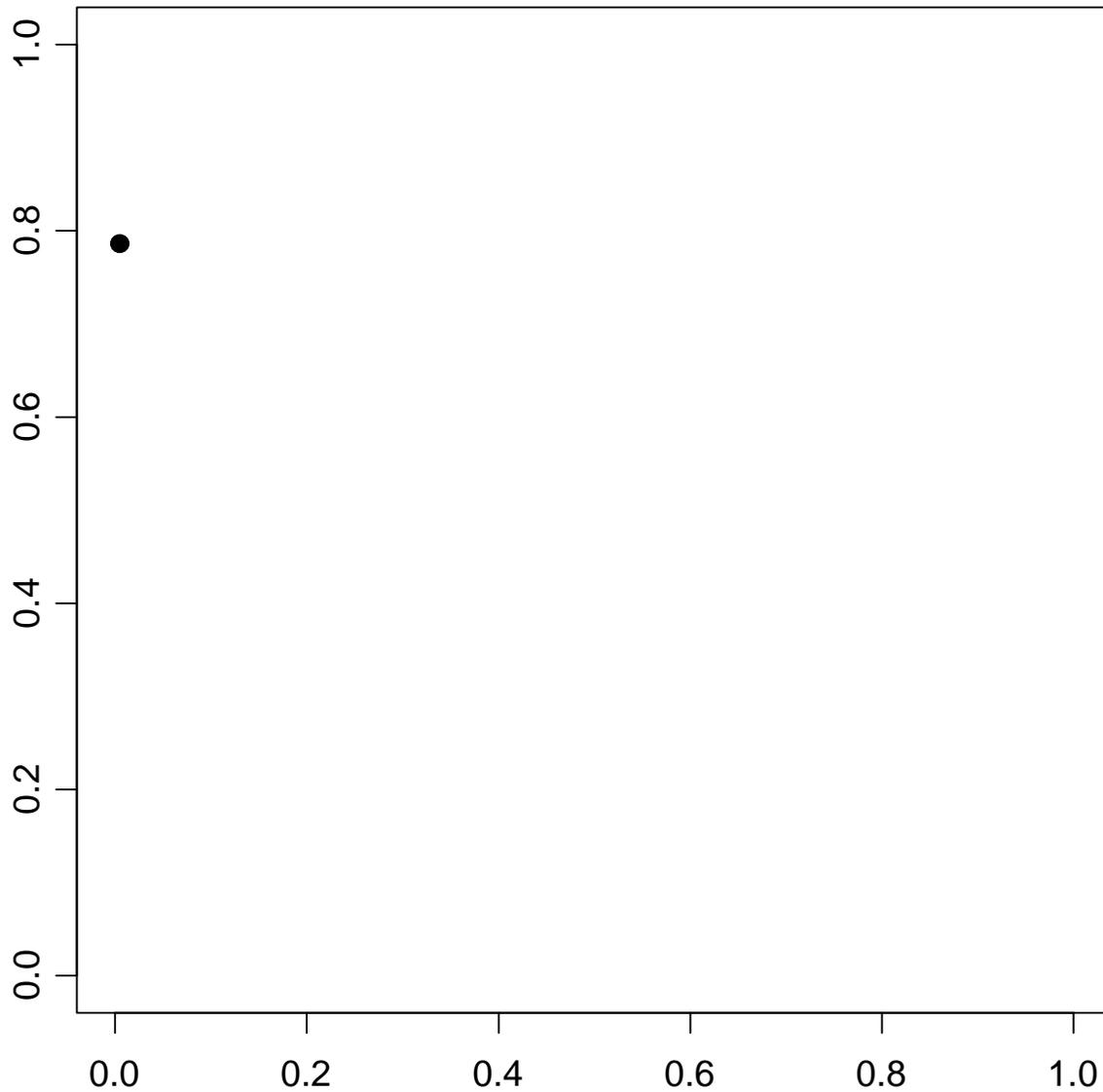
# Two GRTS samples: Size 30

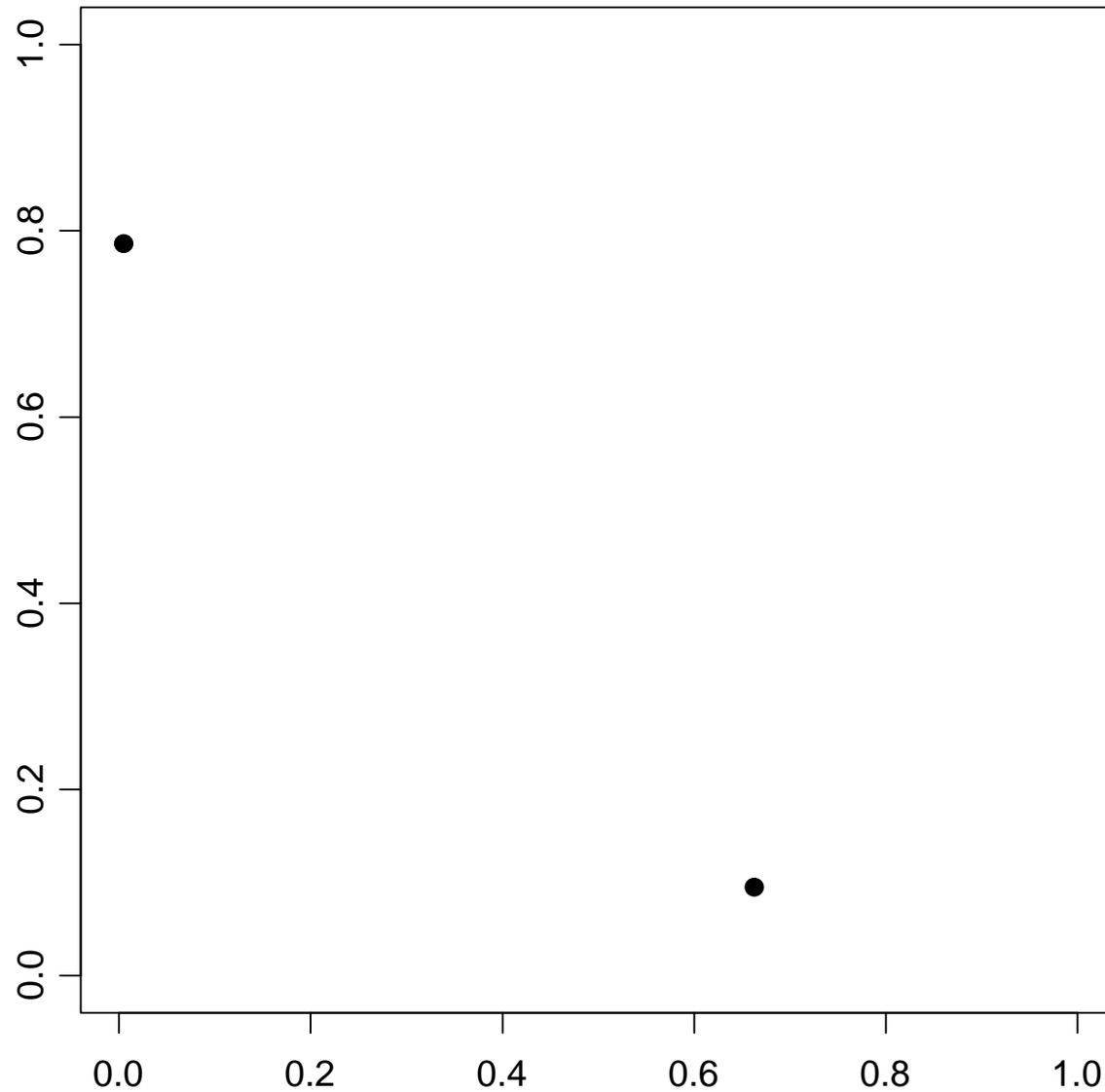
GRTS Sample of 30

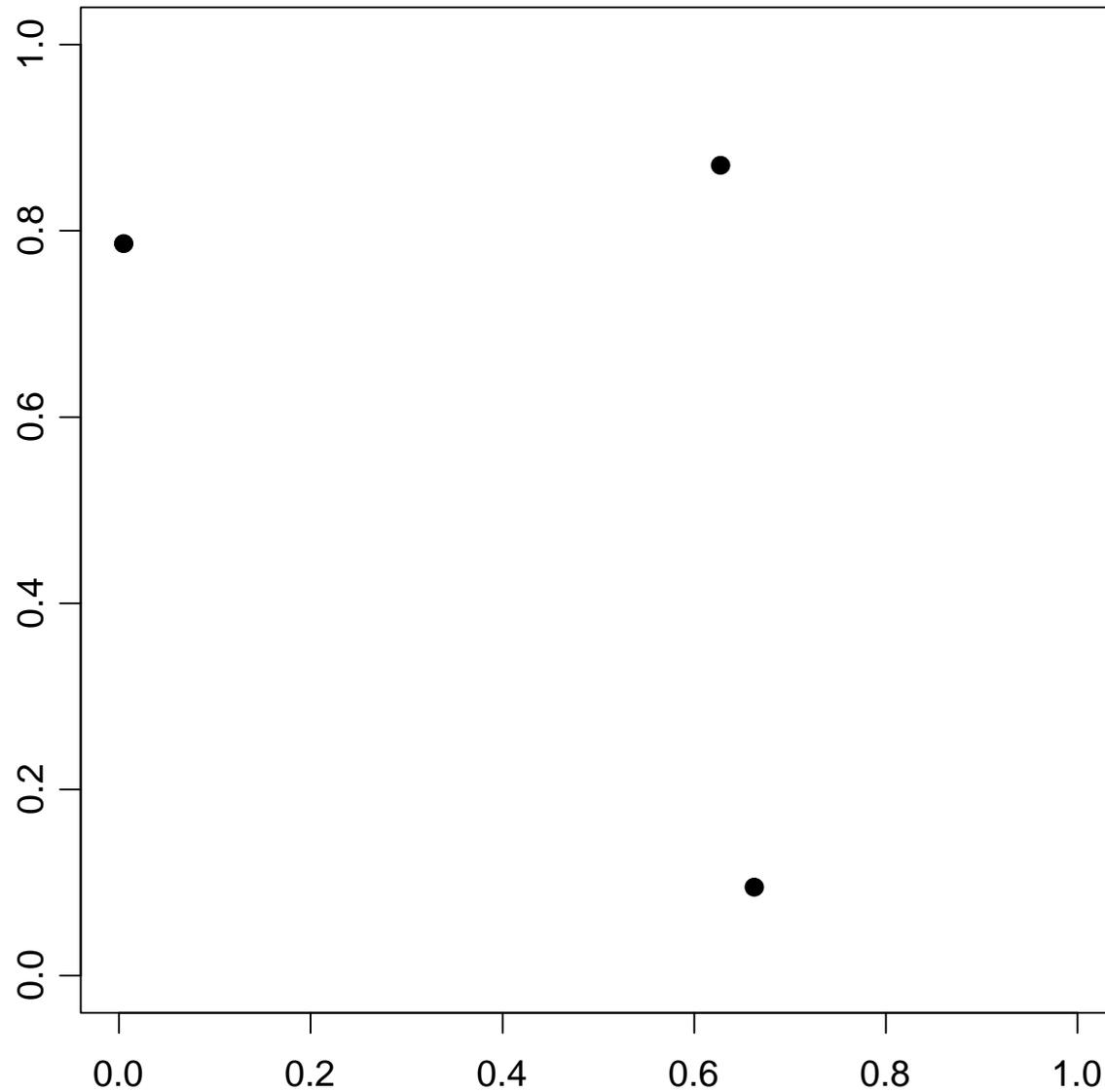


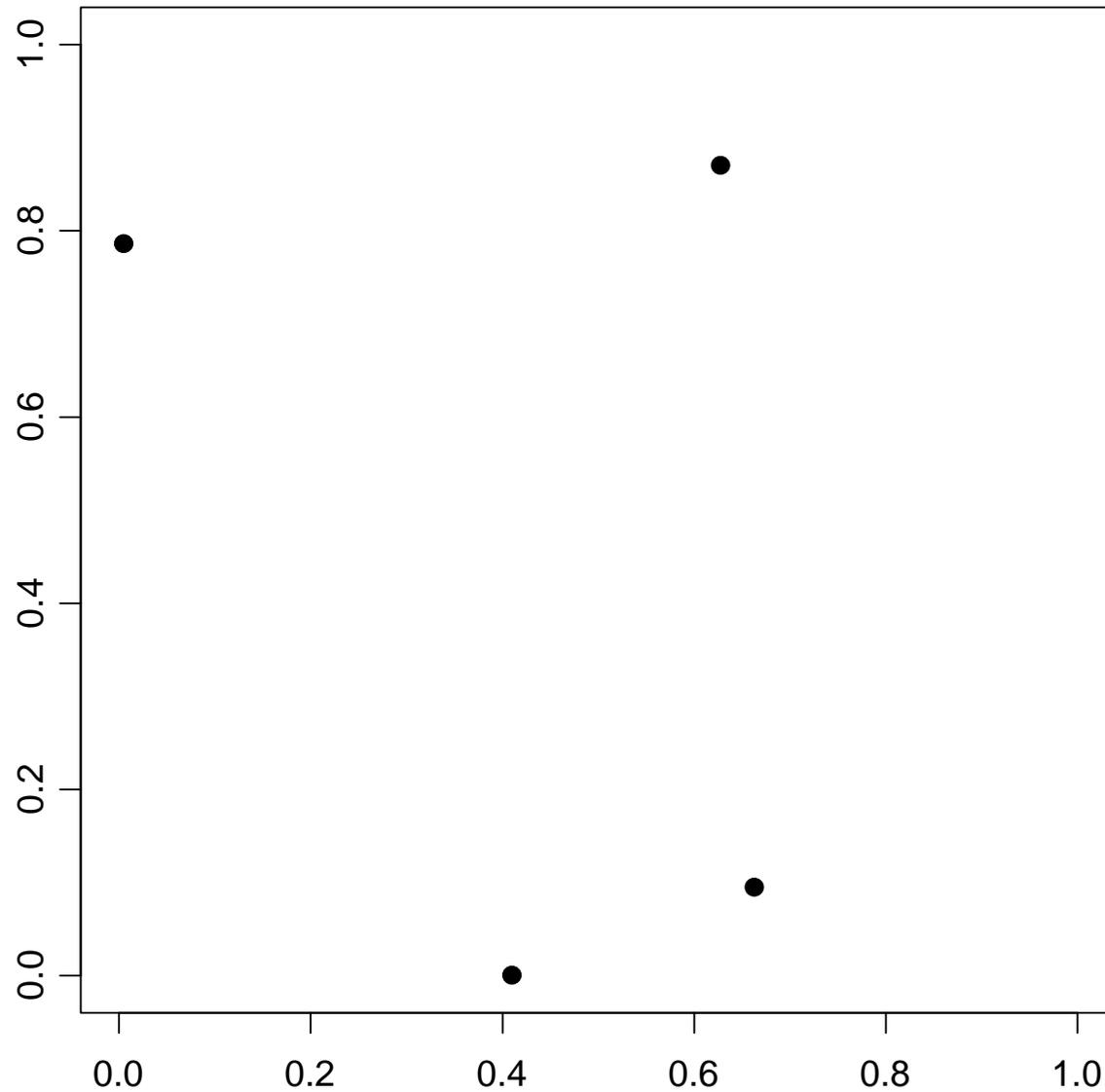
GRTS Sample of 30

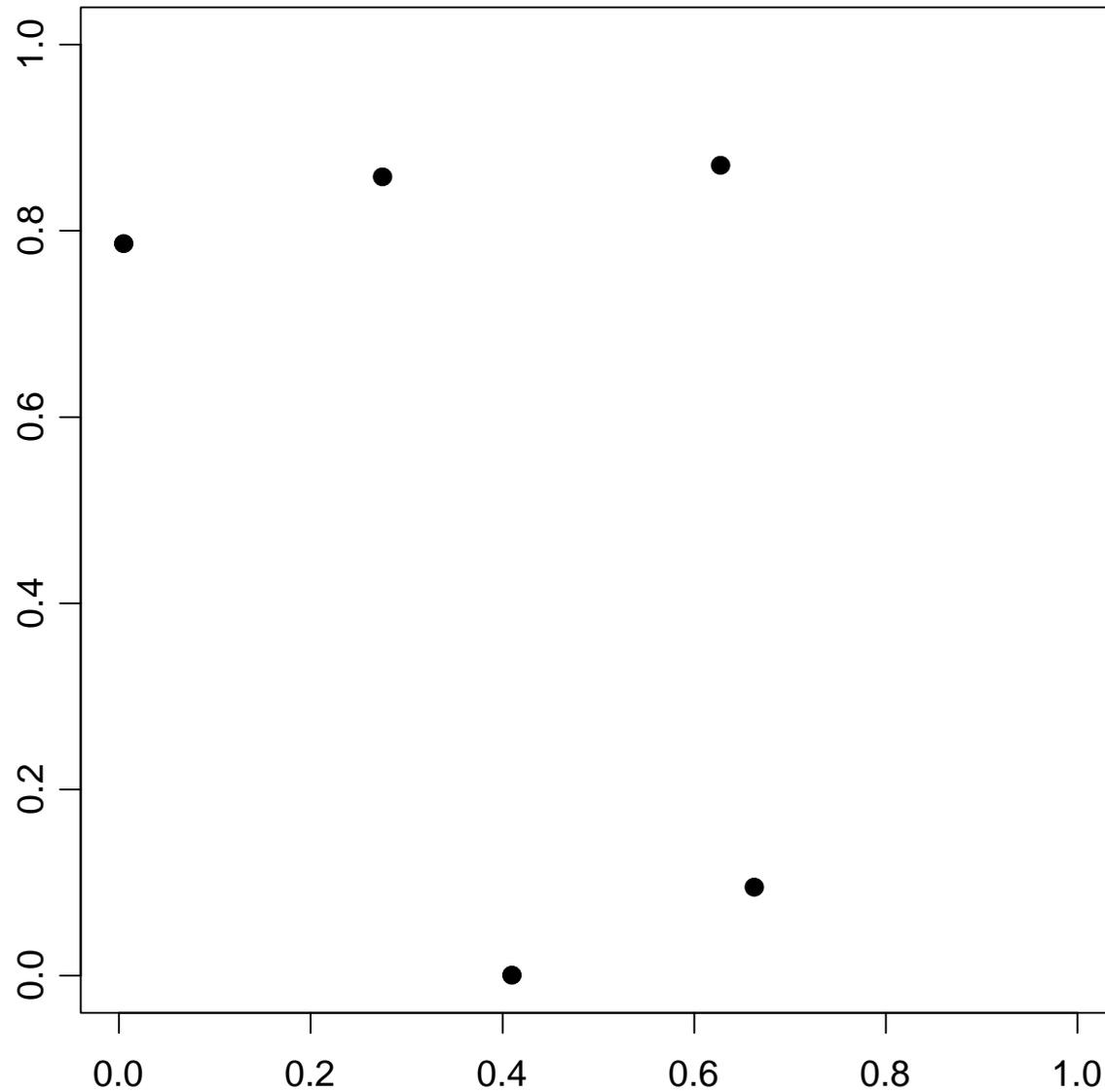


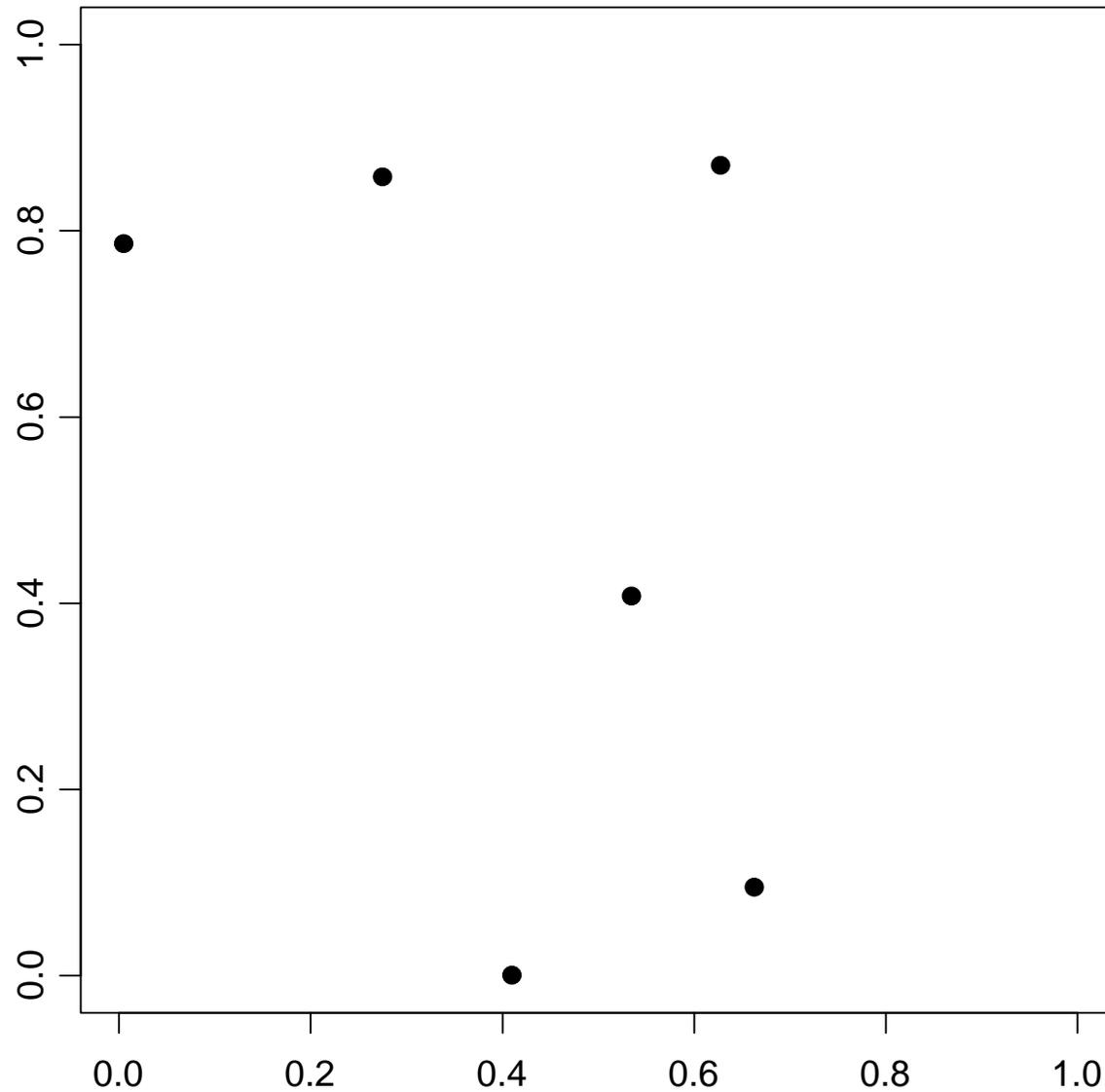


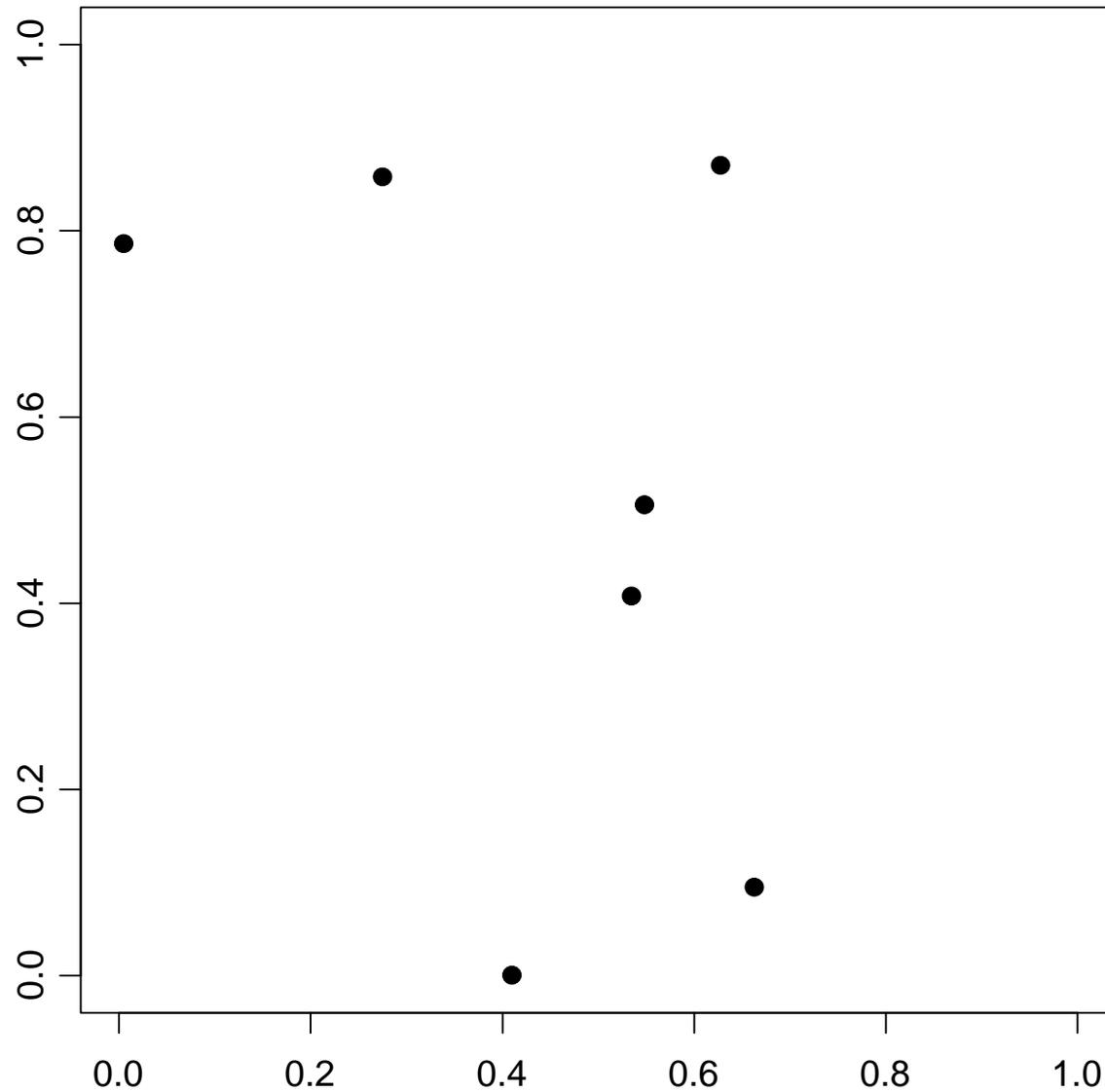


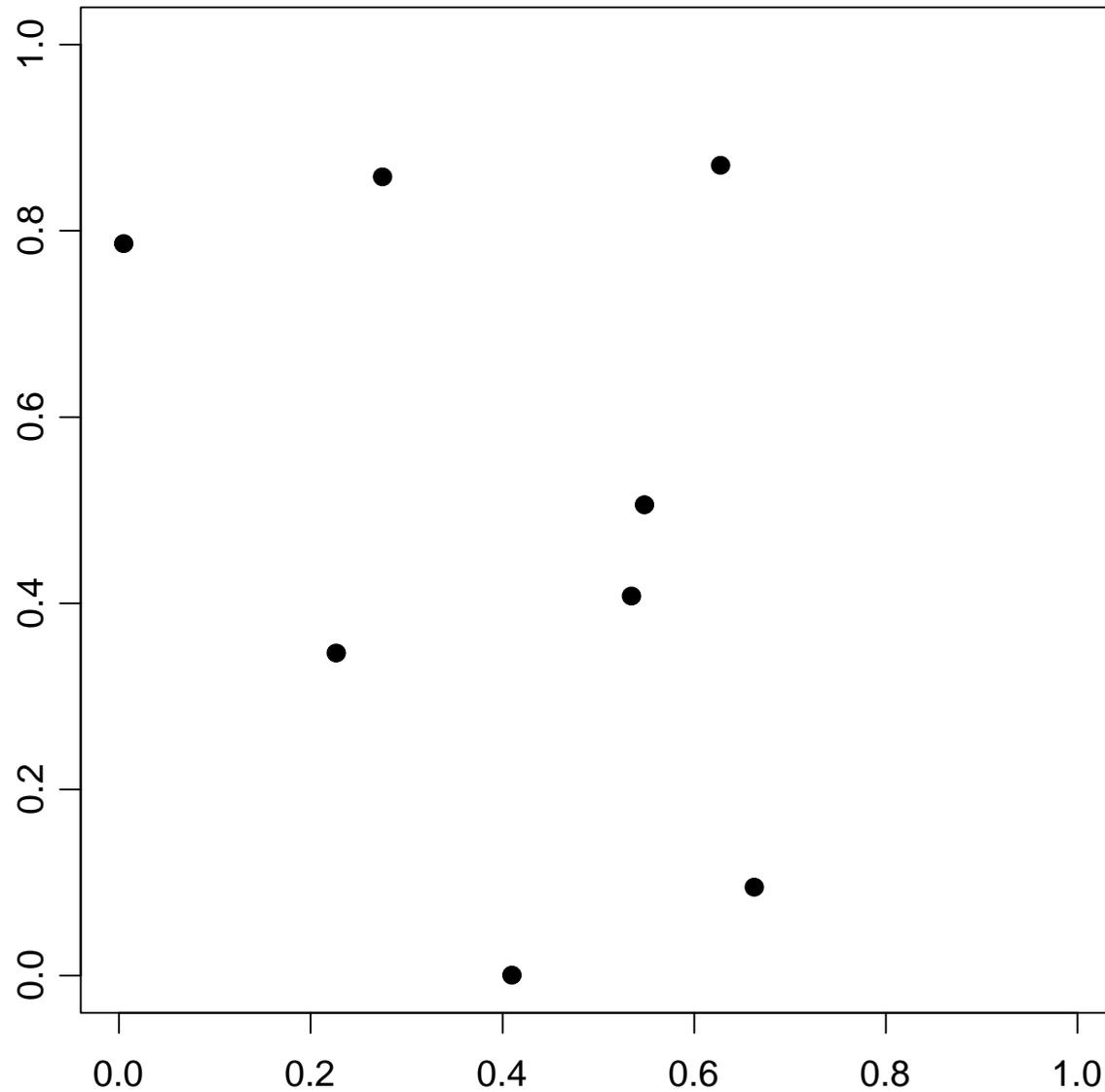


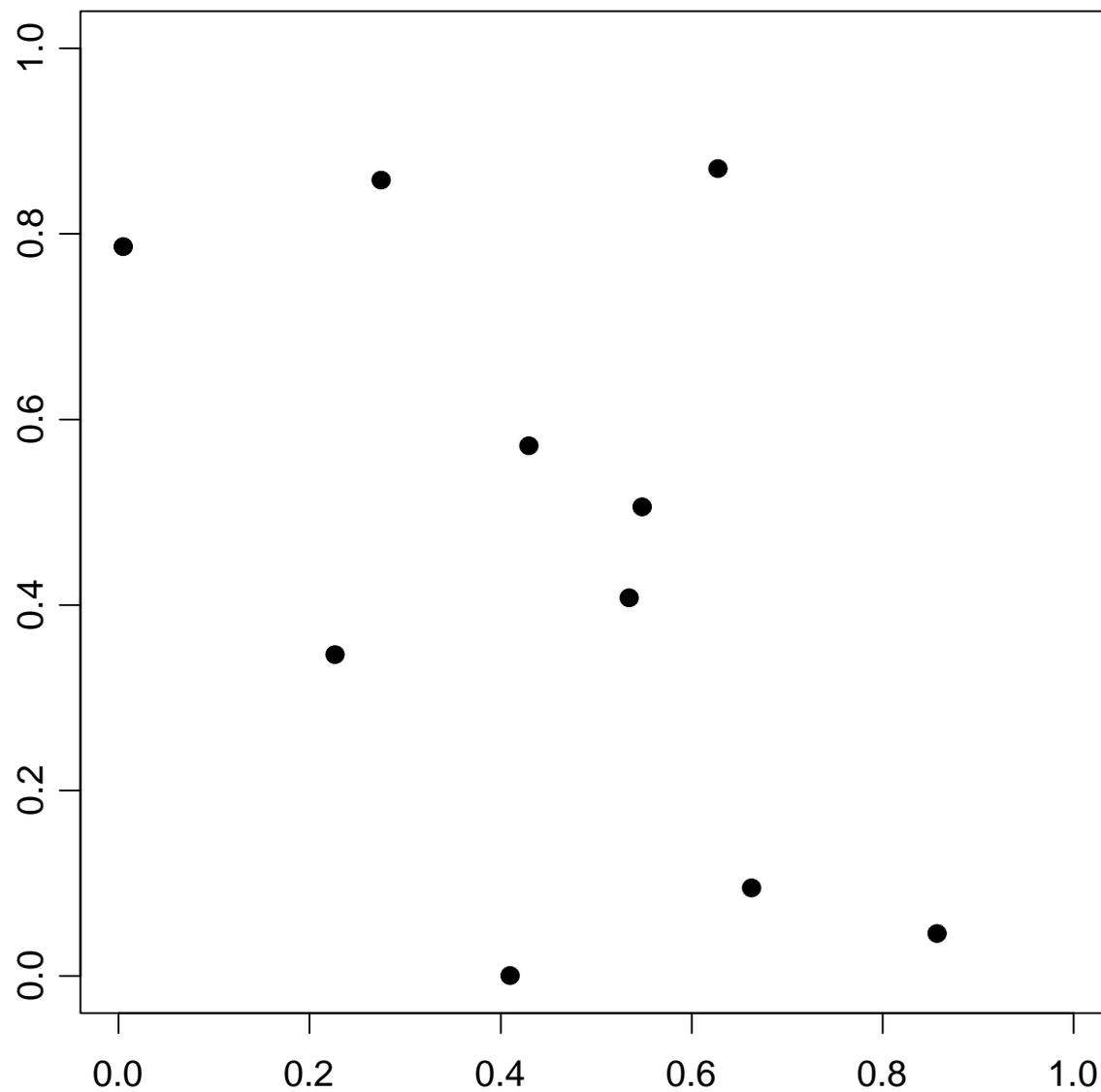


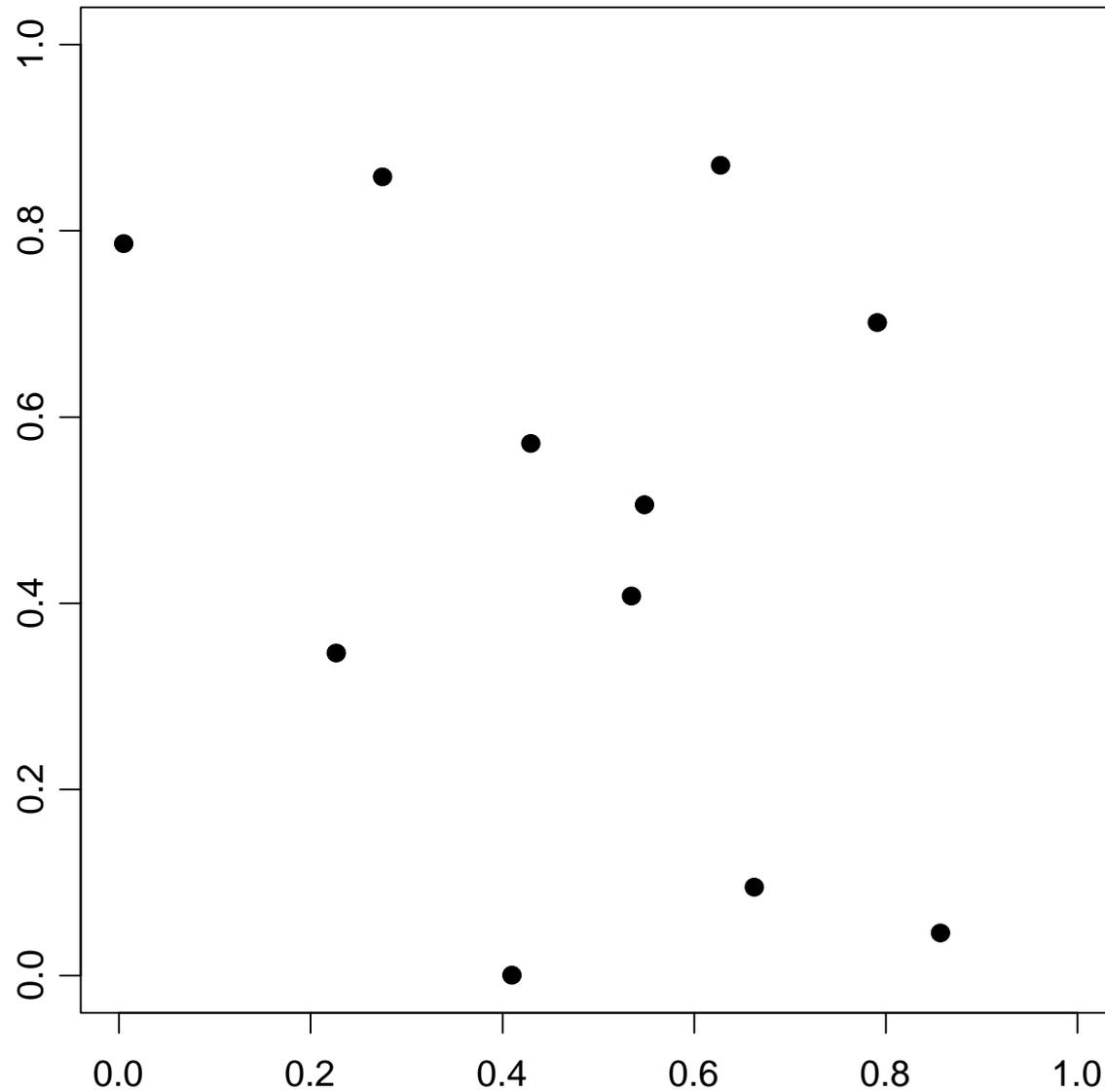


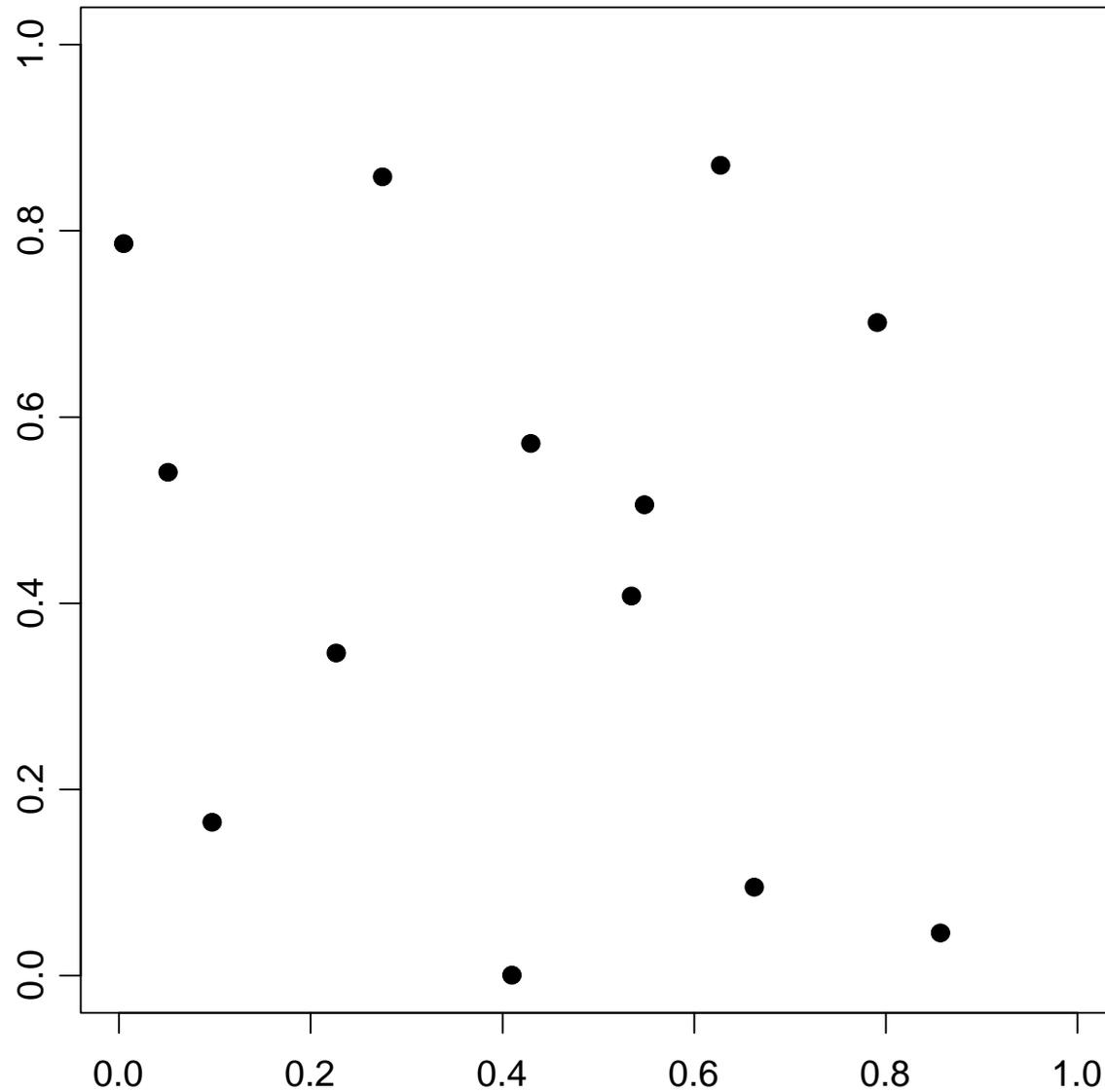


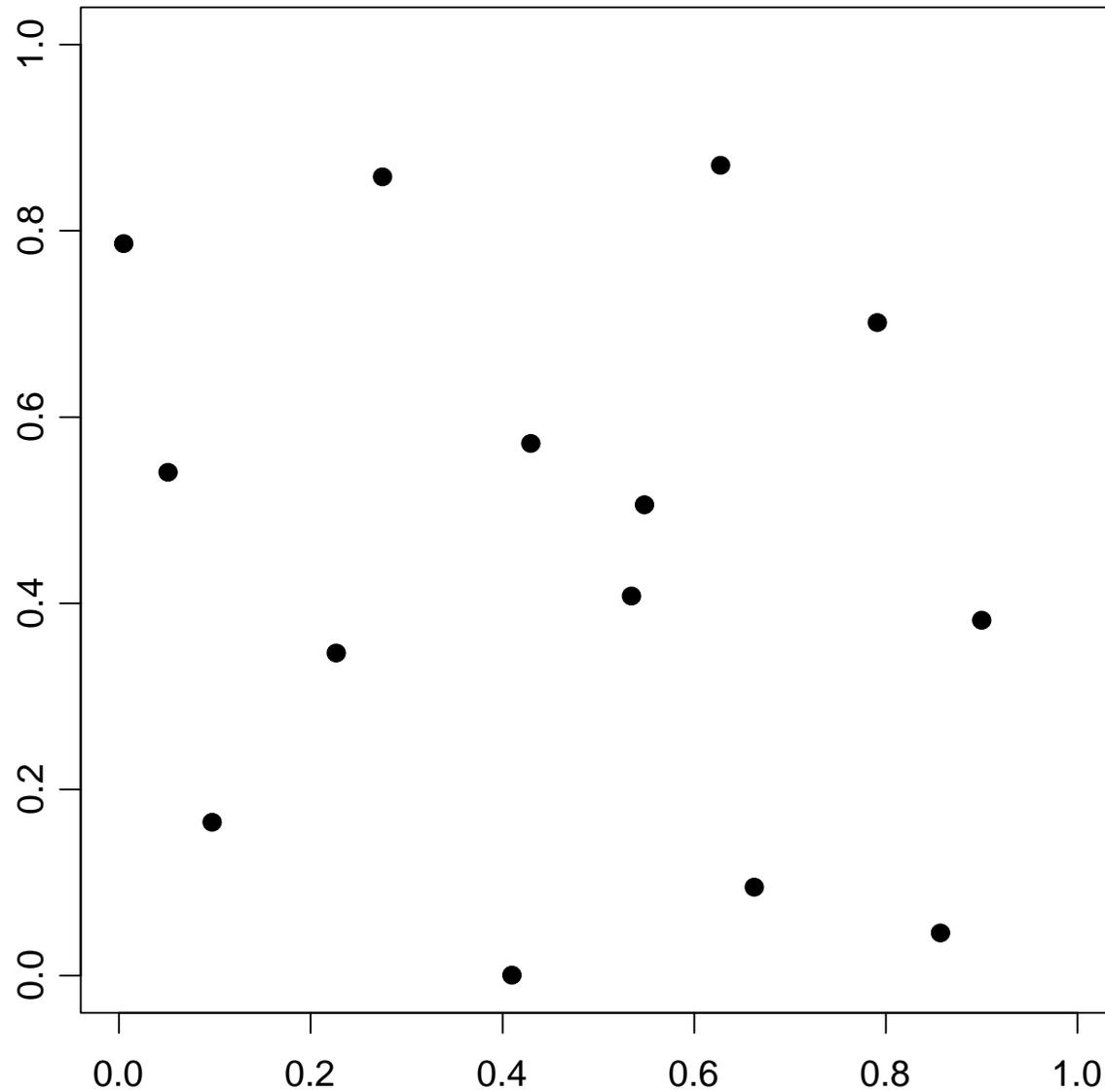


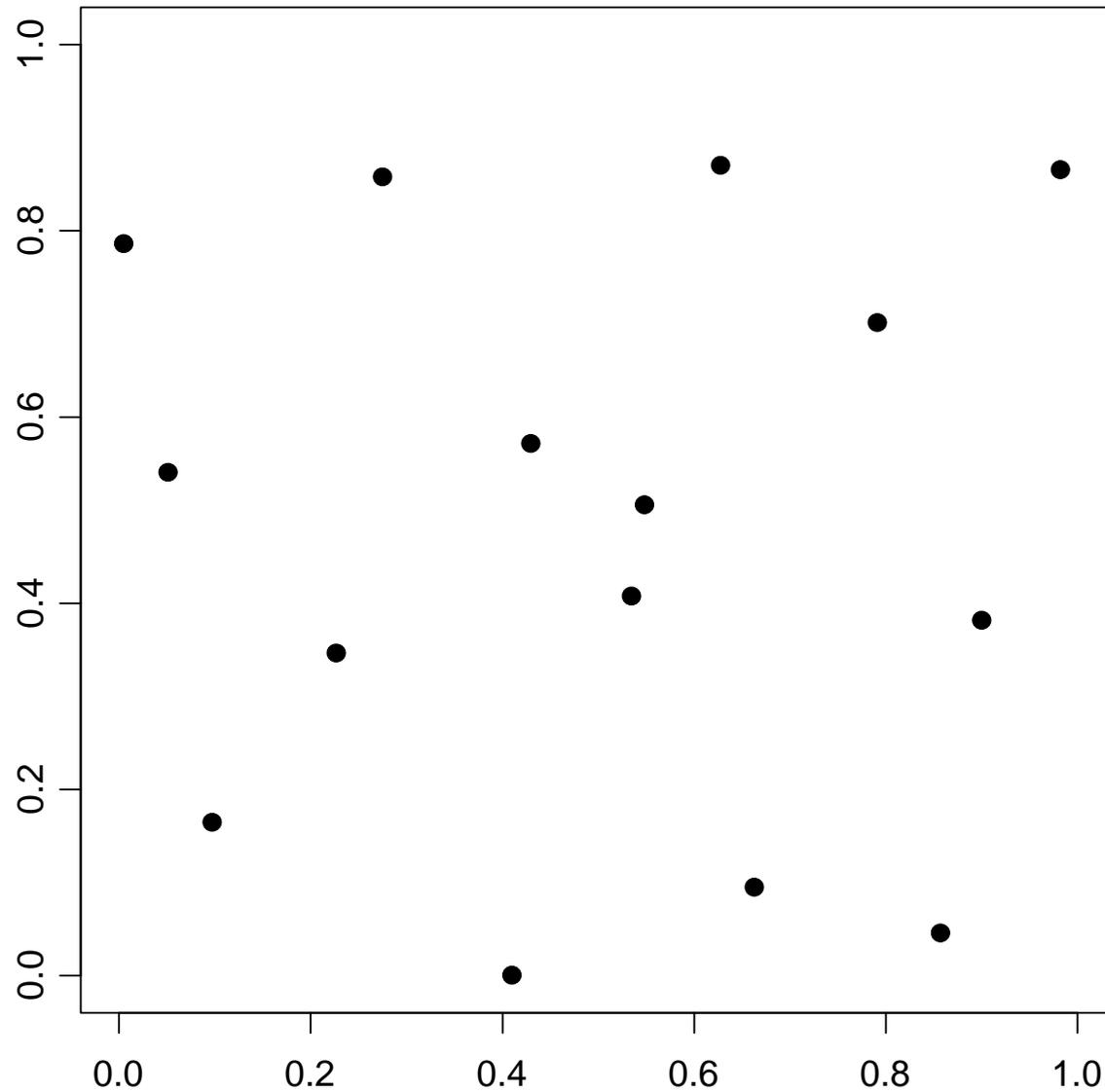


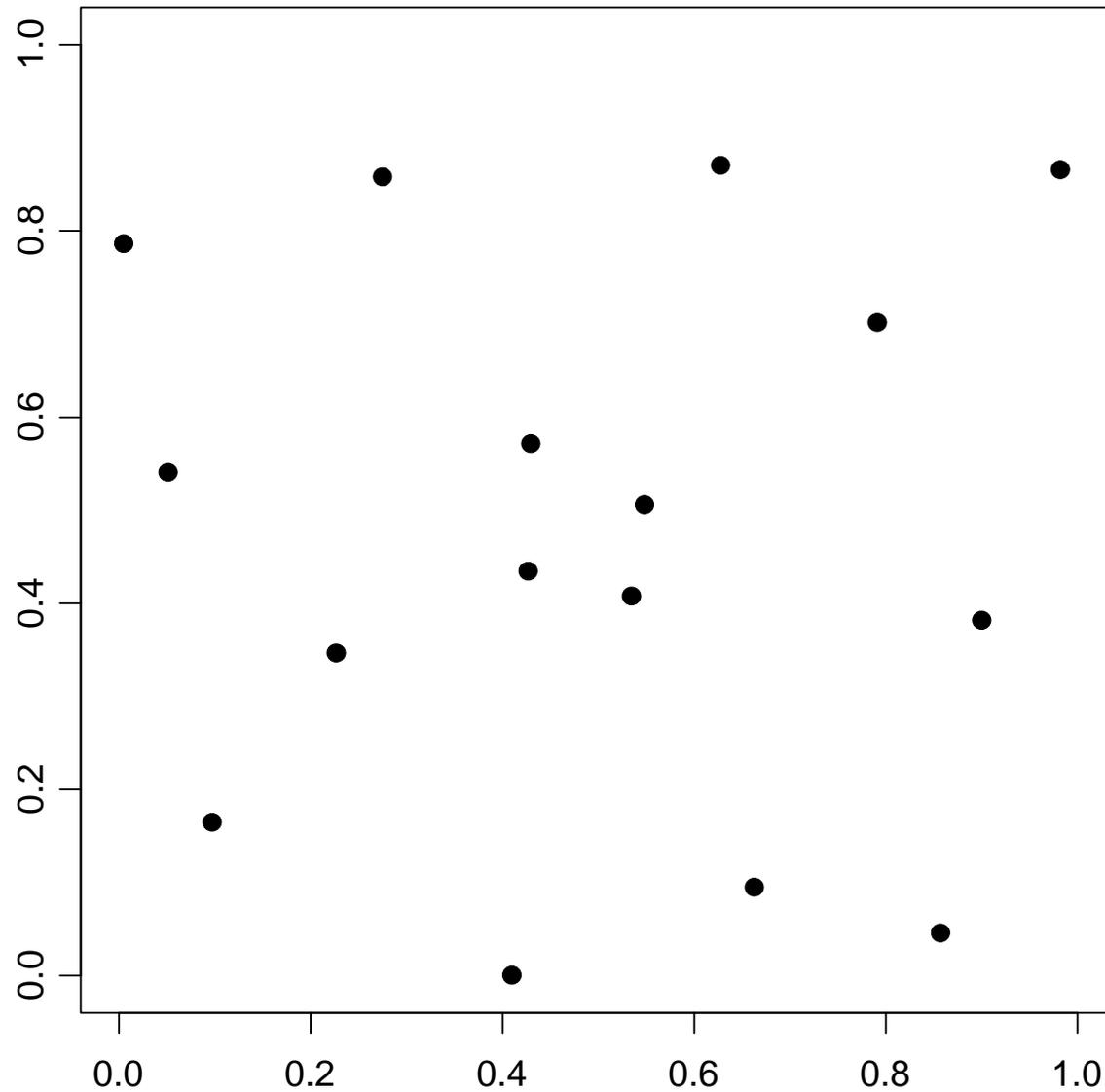


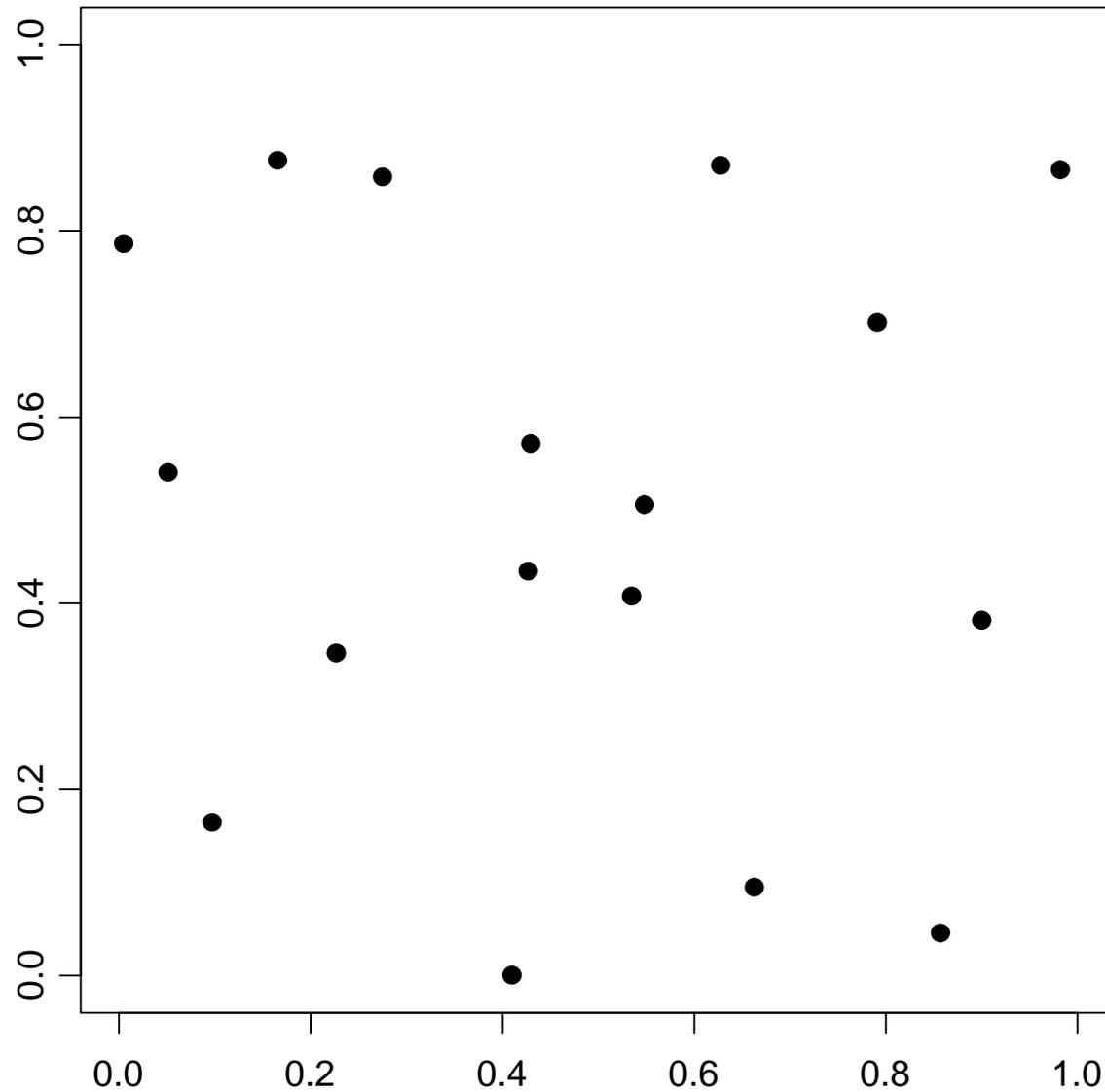


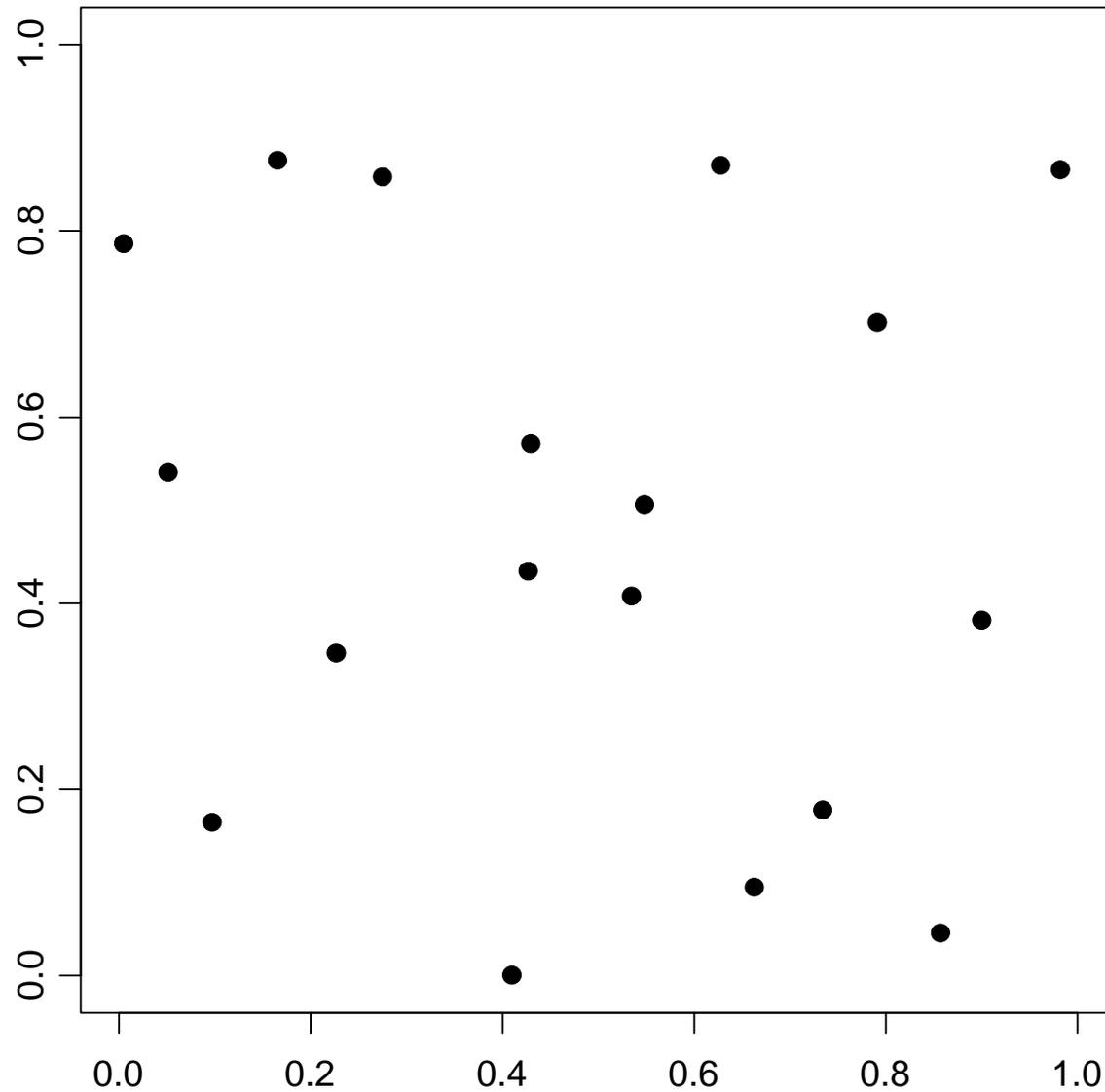


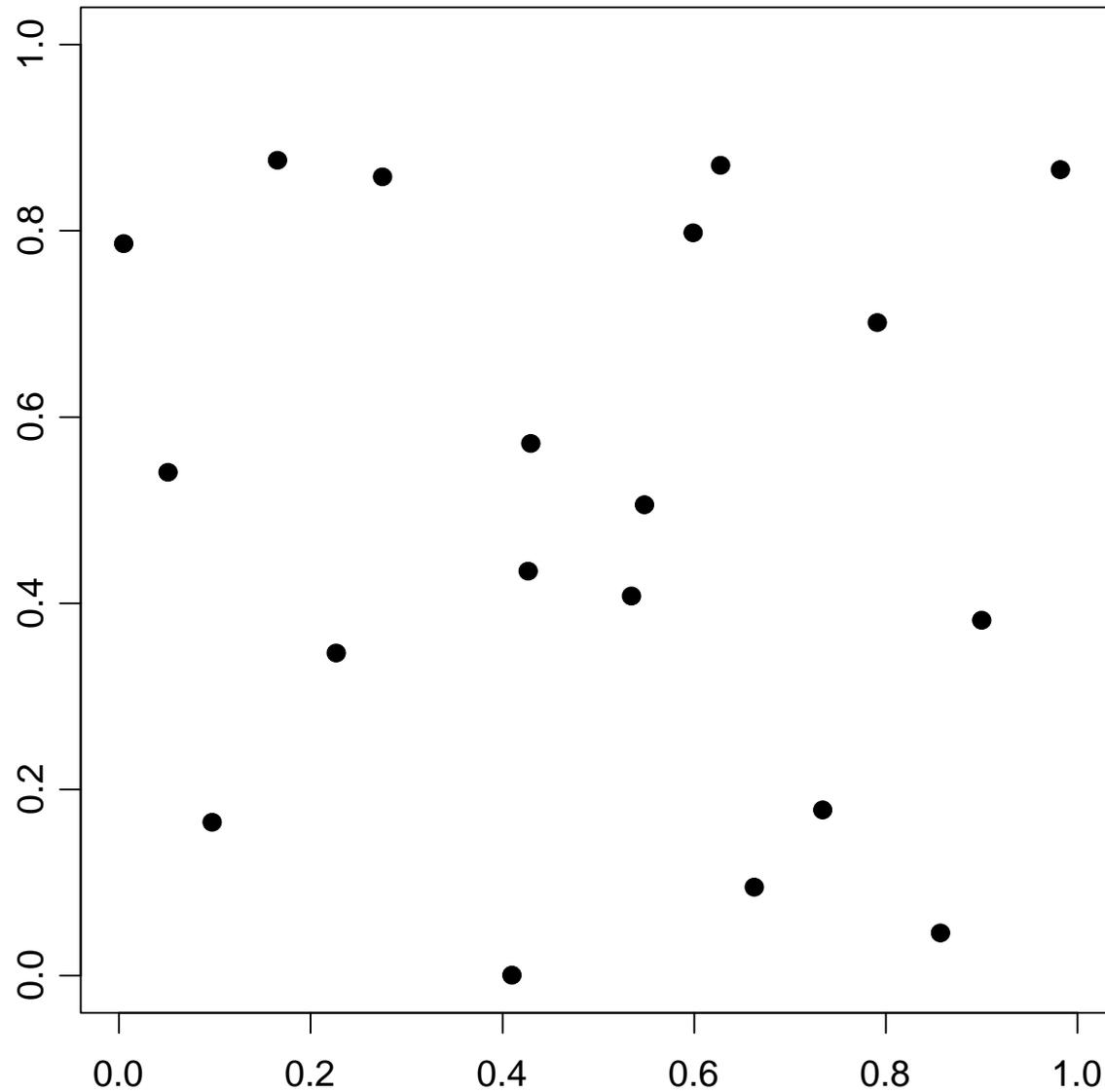


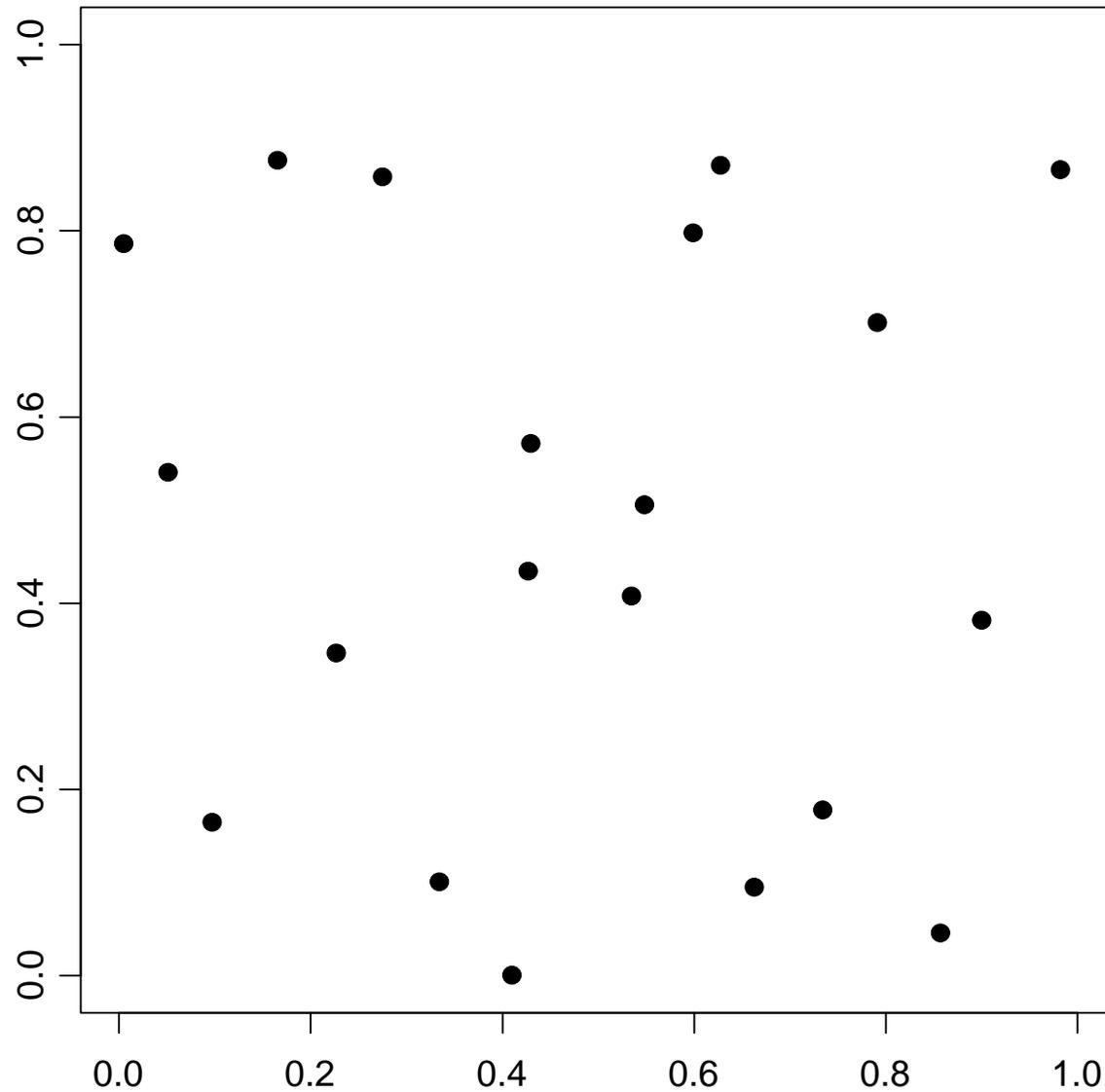


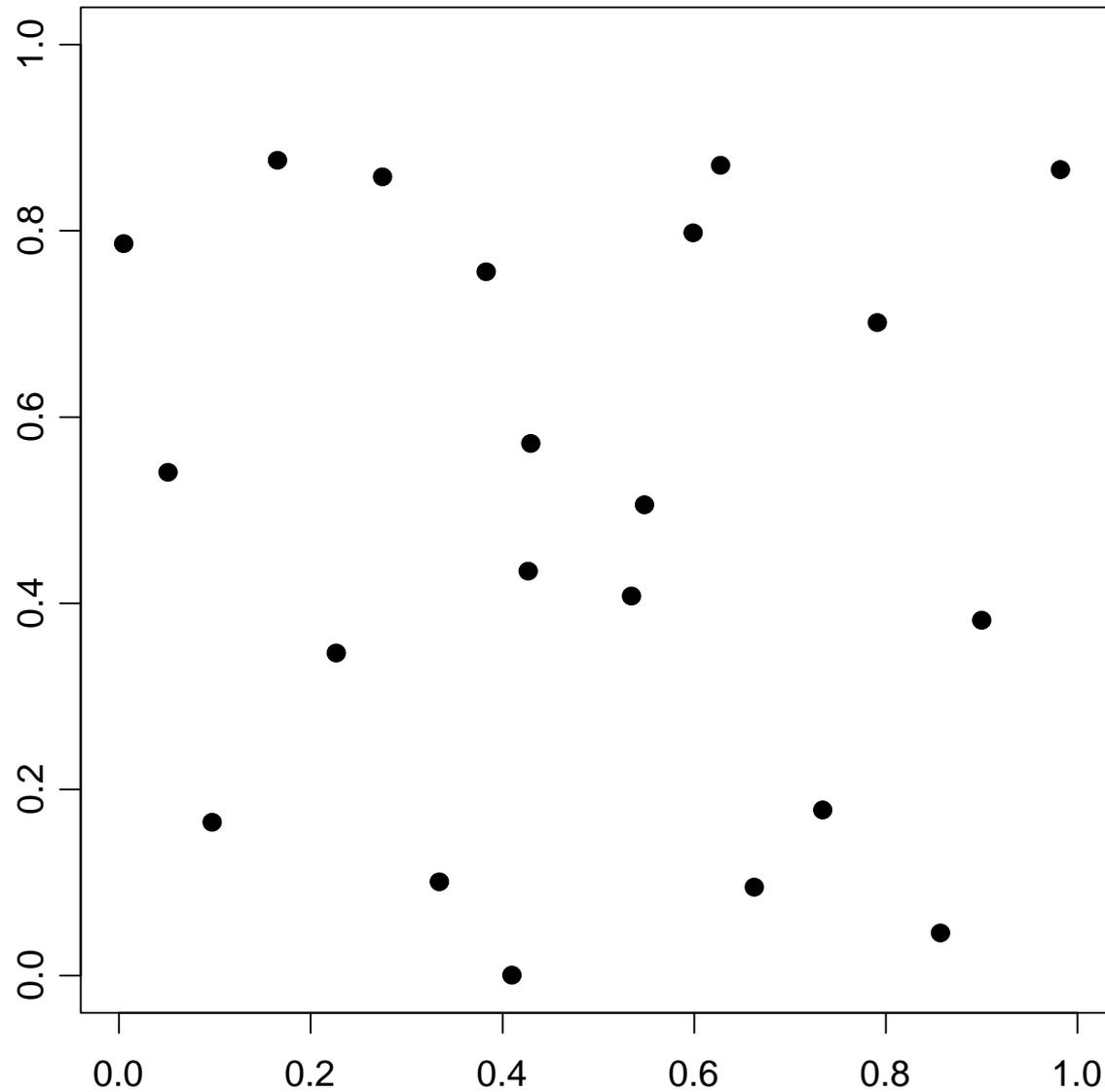


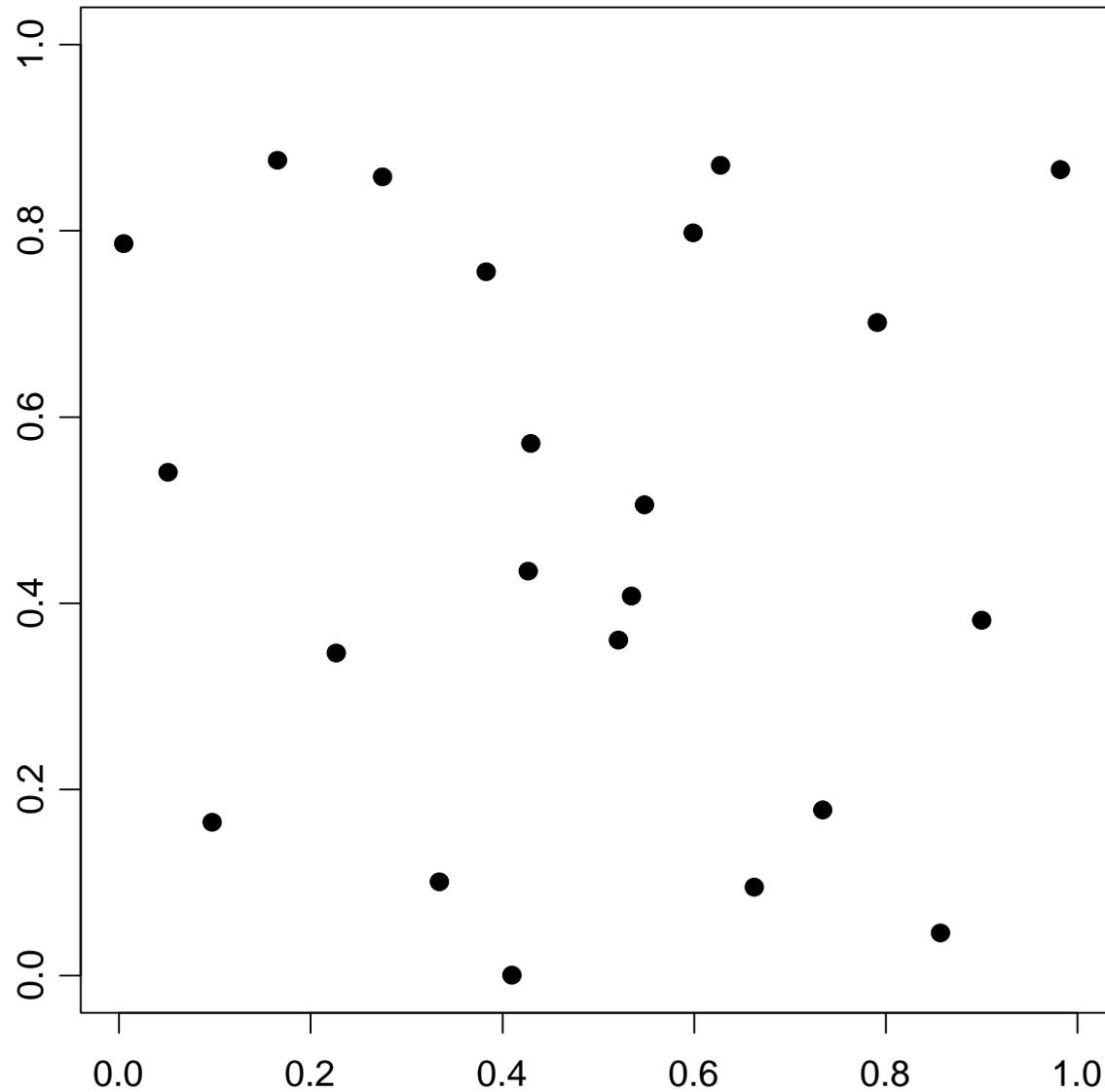


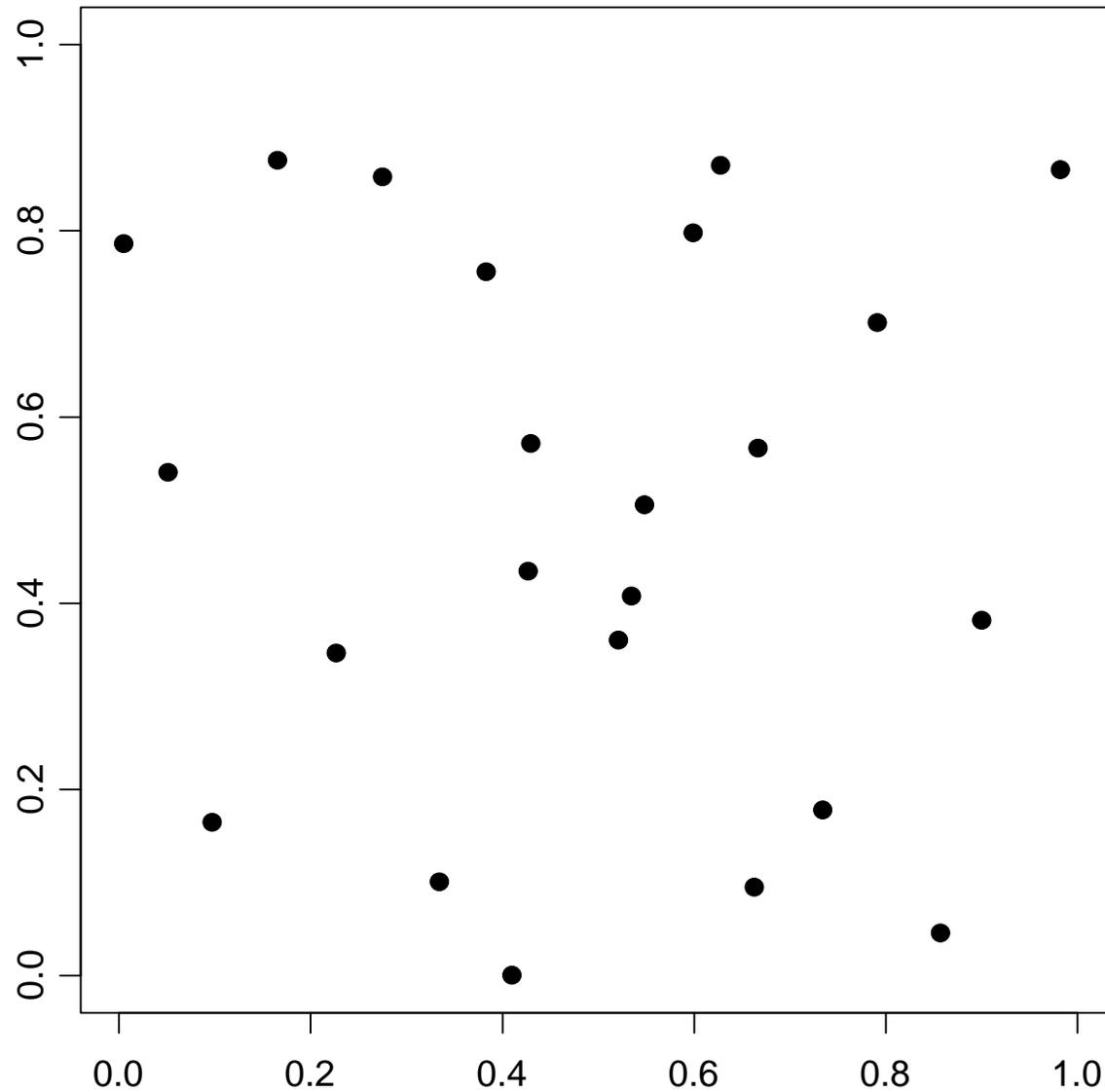


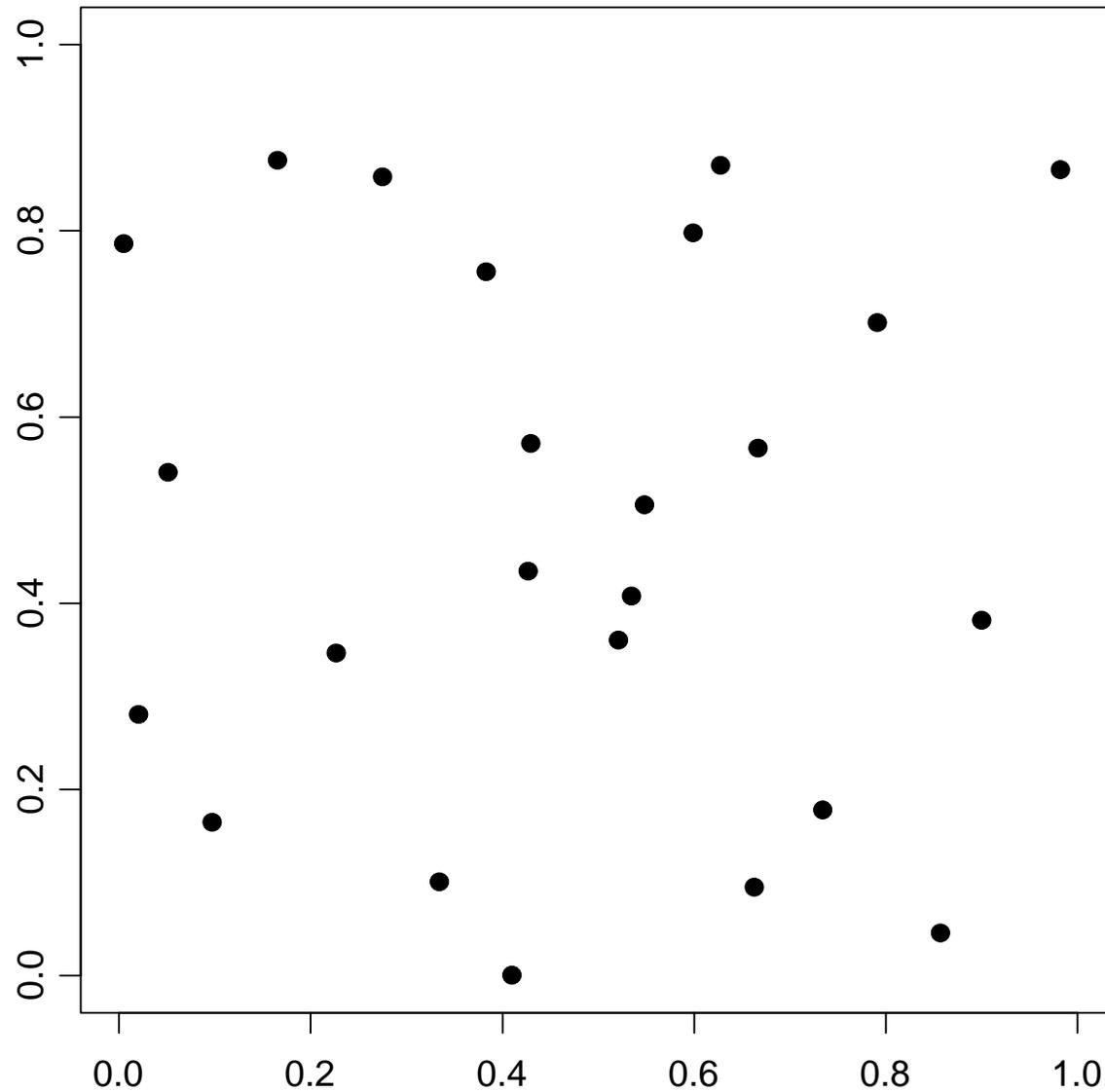


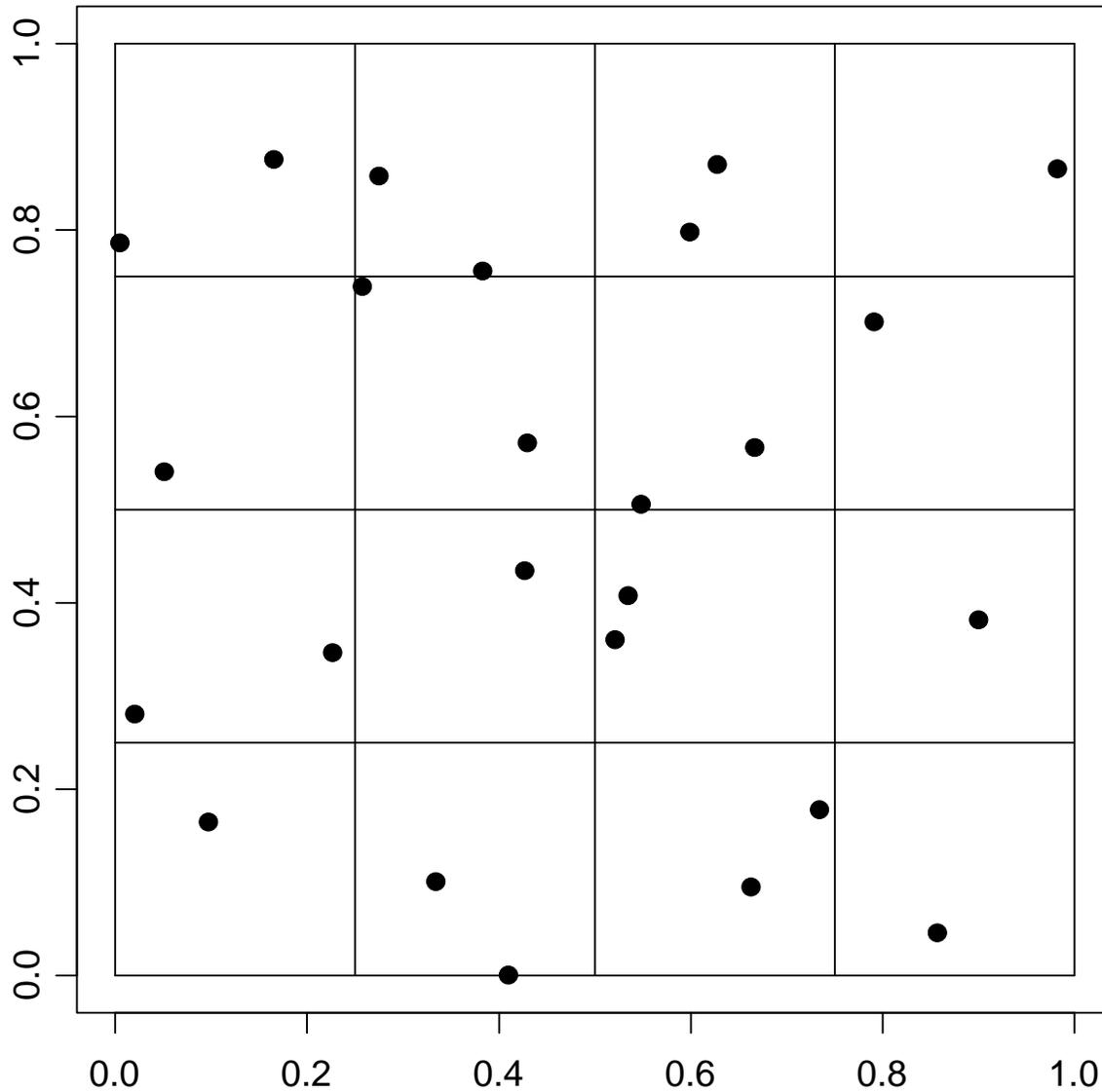


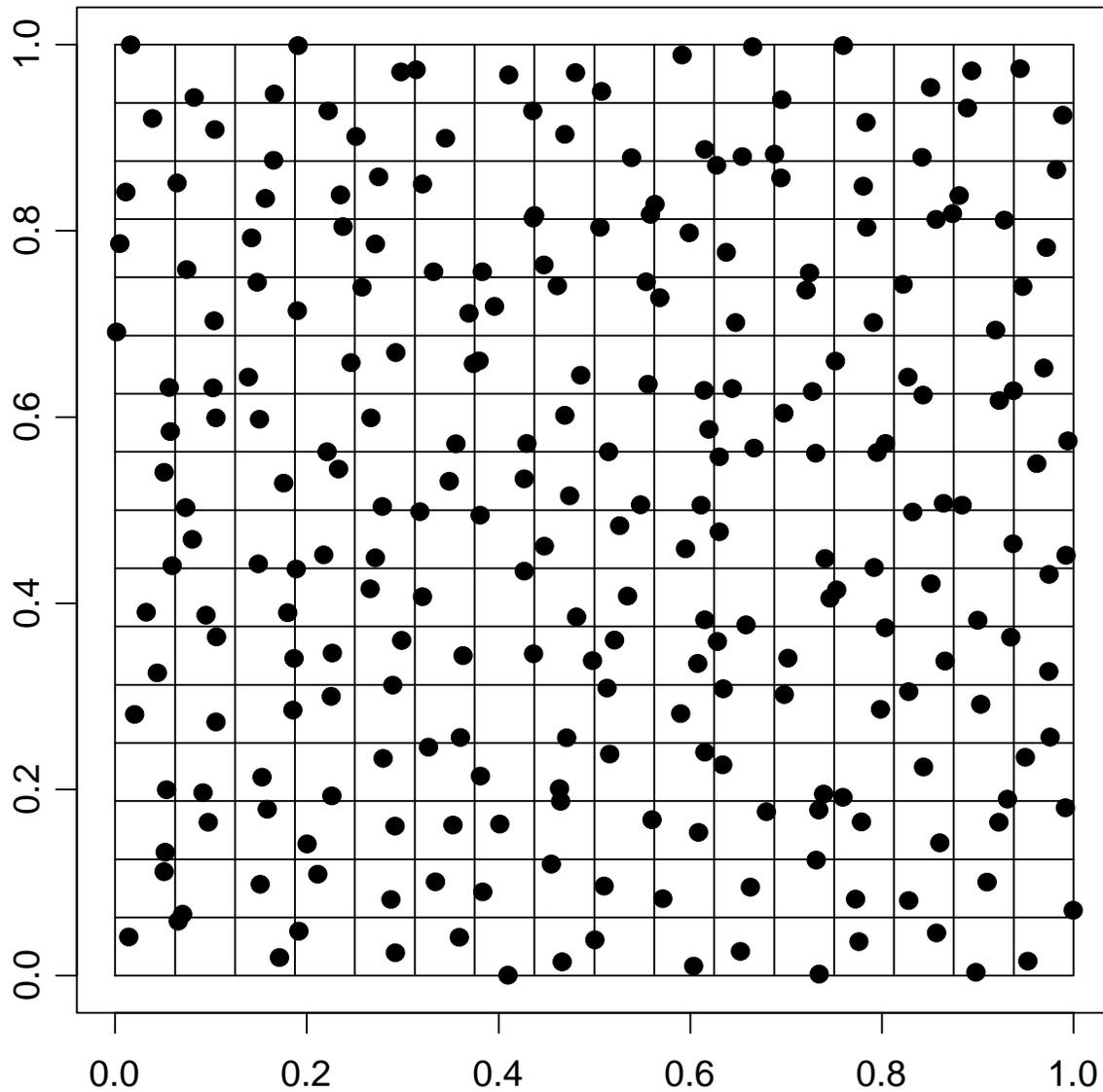








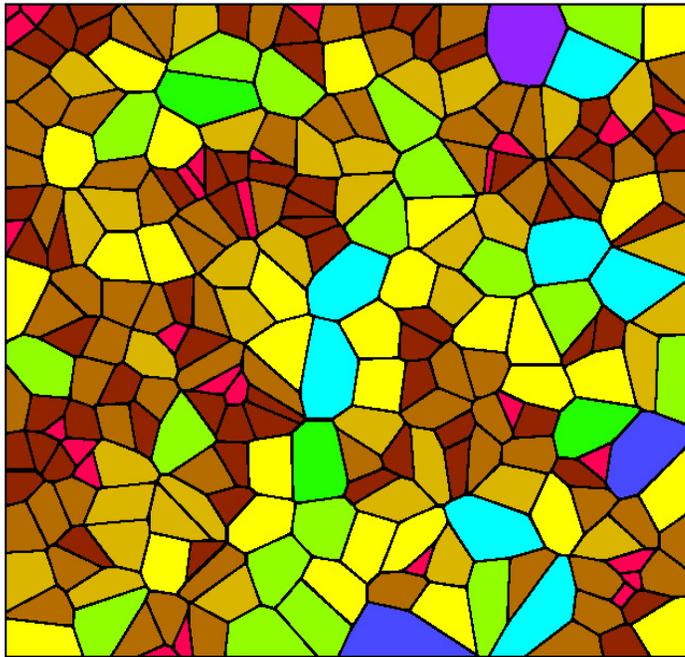




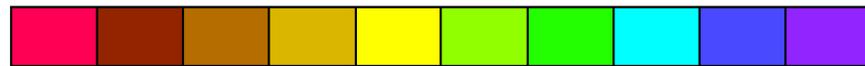
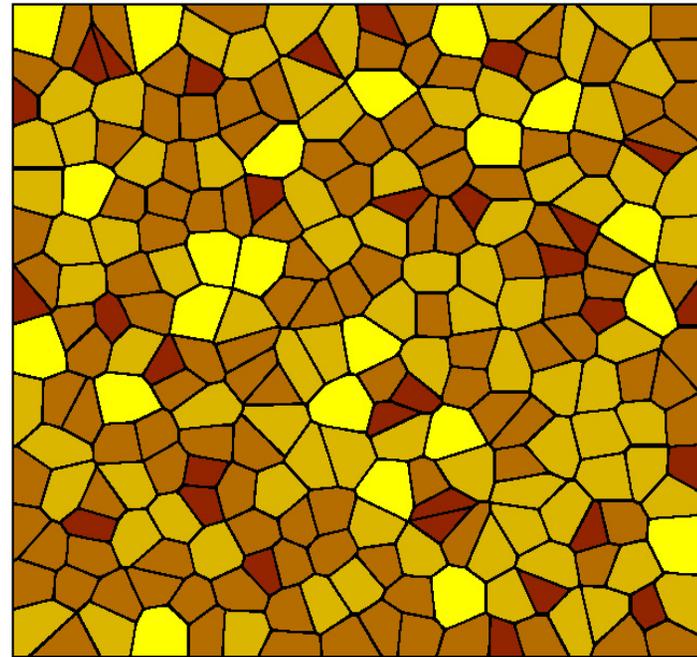
# *Spatial Balance: 256 points*

## Voronoi Polygons

Uniform Sample



GRTS Sample

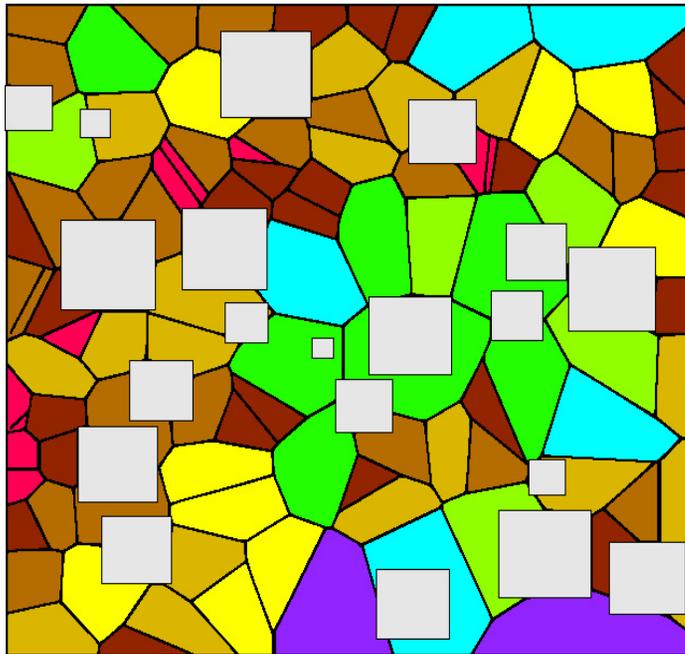


Small ← Polygon Area → Large

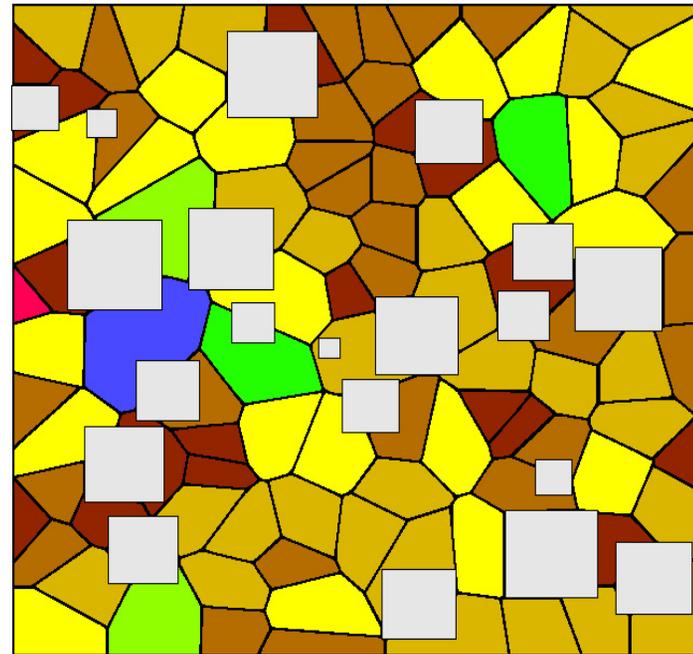
# *Spatial Balance: With oversample*

## Voronoi Polygons

Uniform Sample

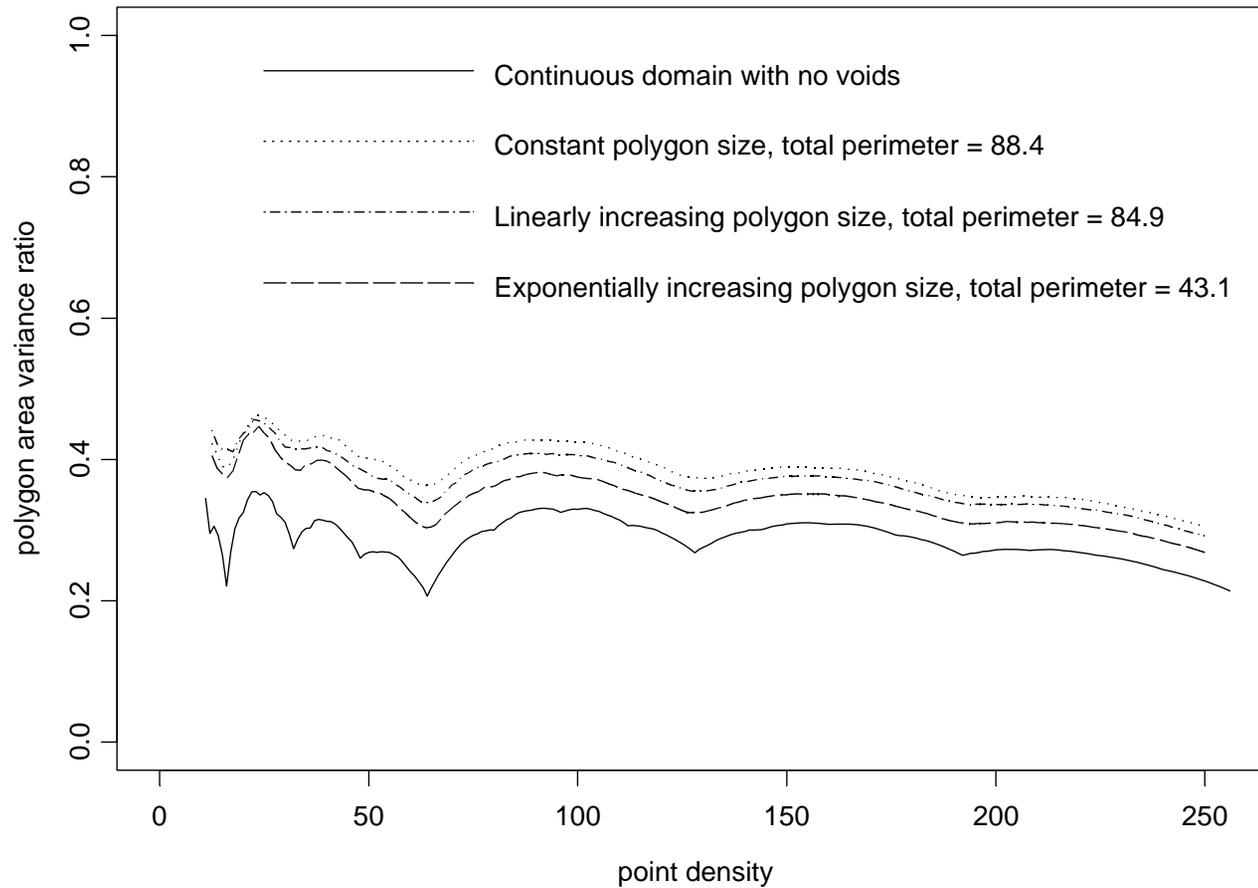


GRTS Sample

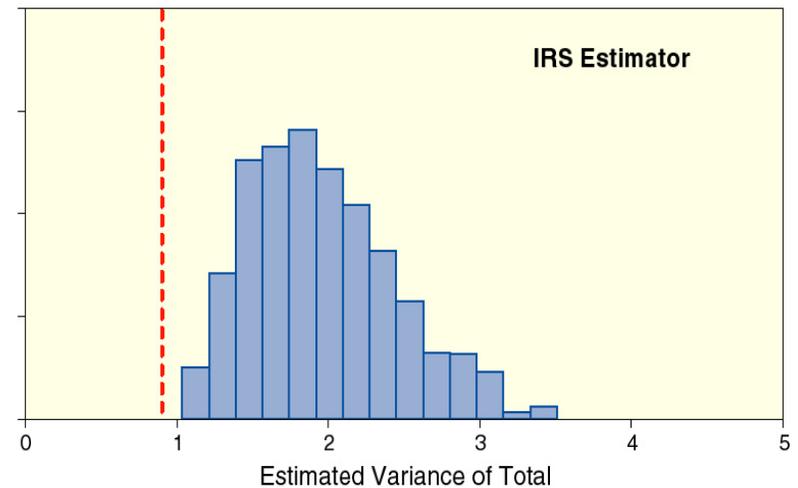
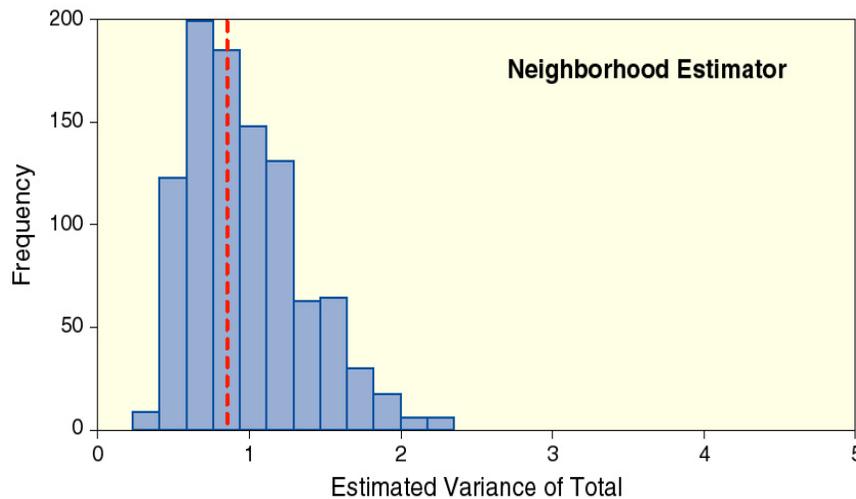


Small ← Polygon Area → Large

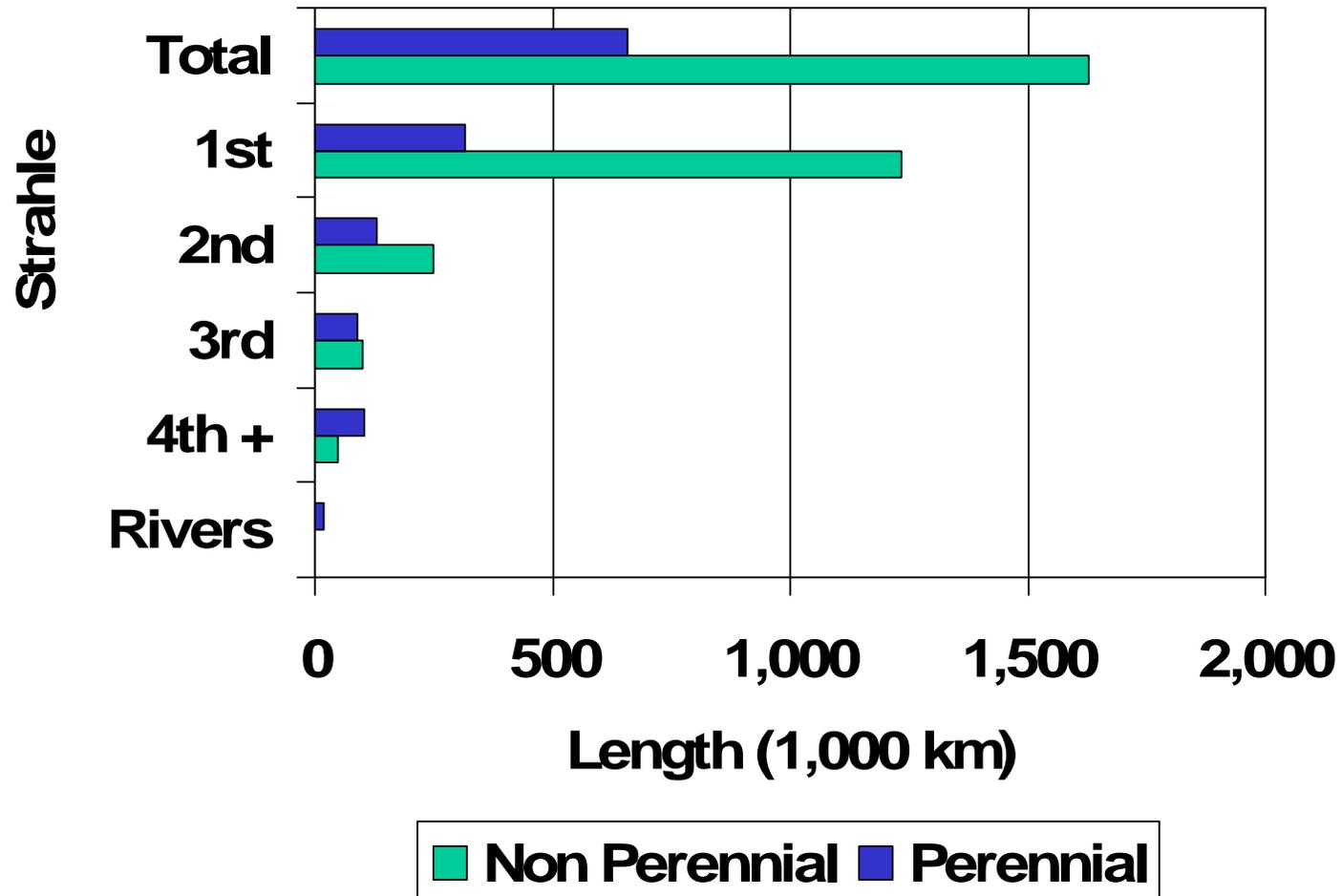
# Ratio of GRTS to SRS Voronoi polygon size variance



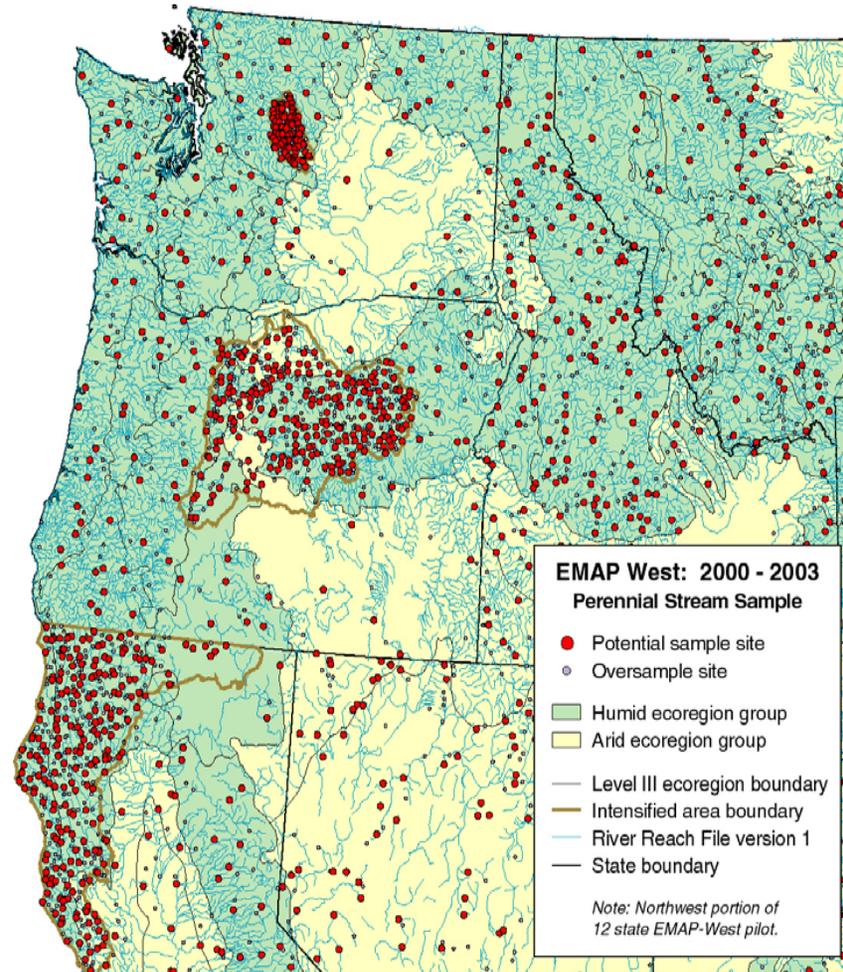
# *Impact on Variance Estimators of Totals*

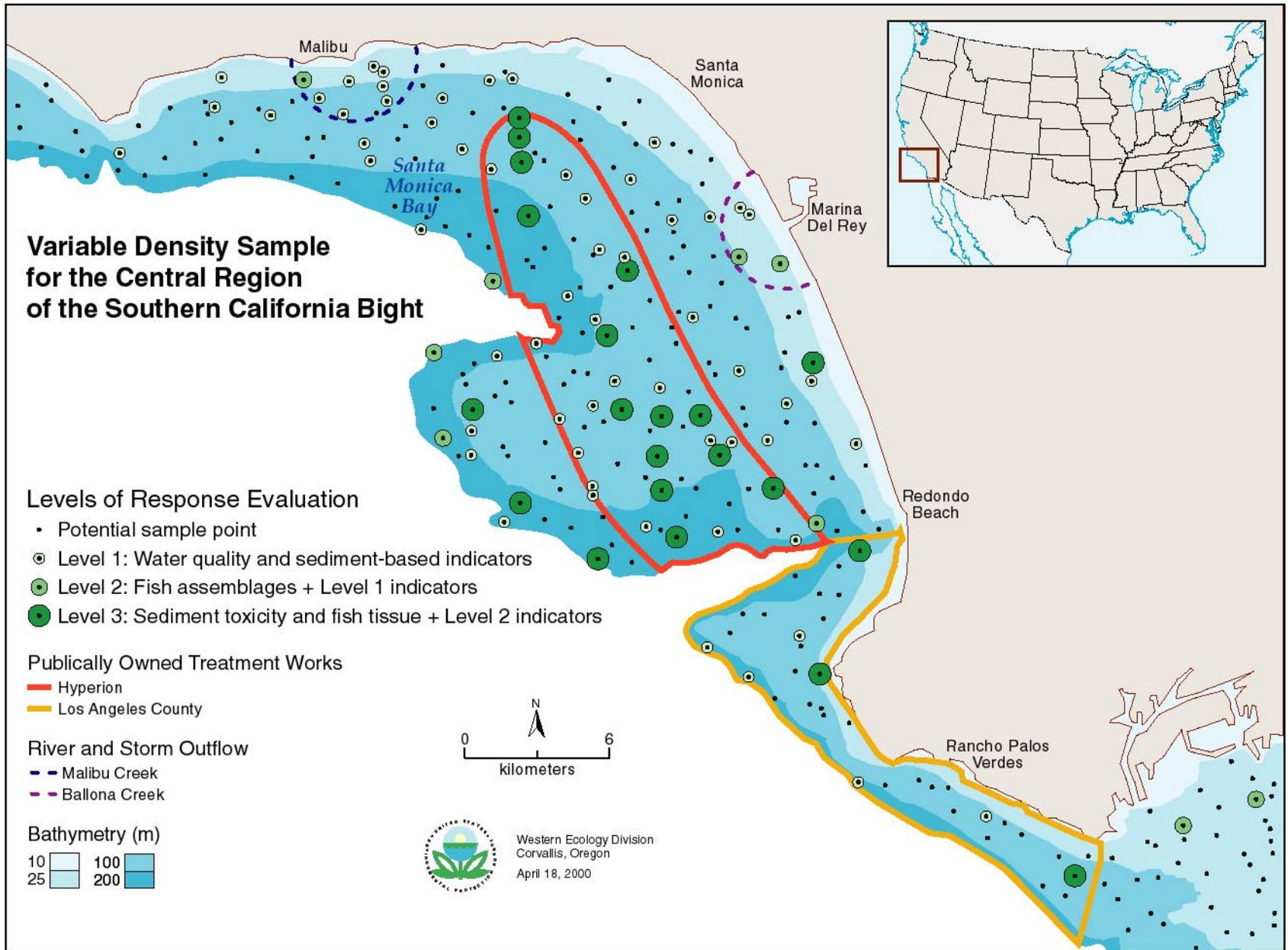


## *RF3 Stream Length: EMAP West*



# *Perennial Streams GRTS sample*





projects/emapgis/urquhart/california\_bight/figure12-4.ai  
4/18/00 smp

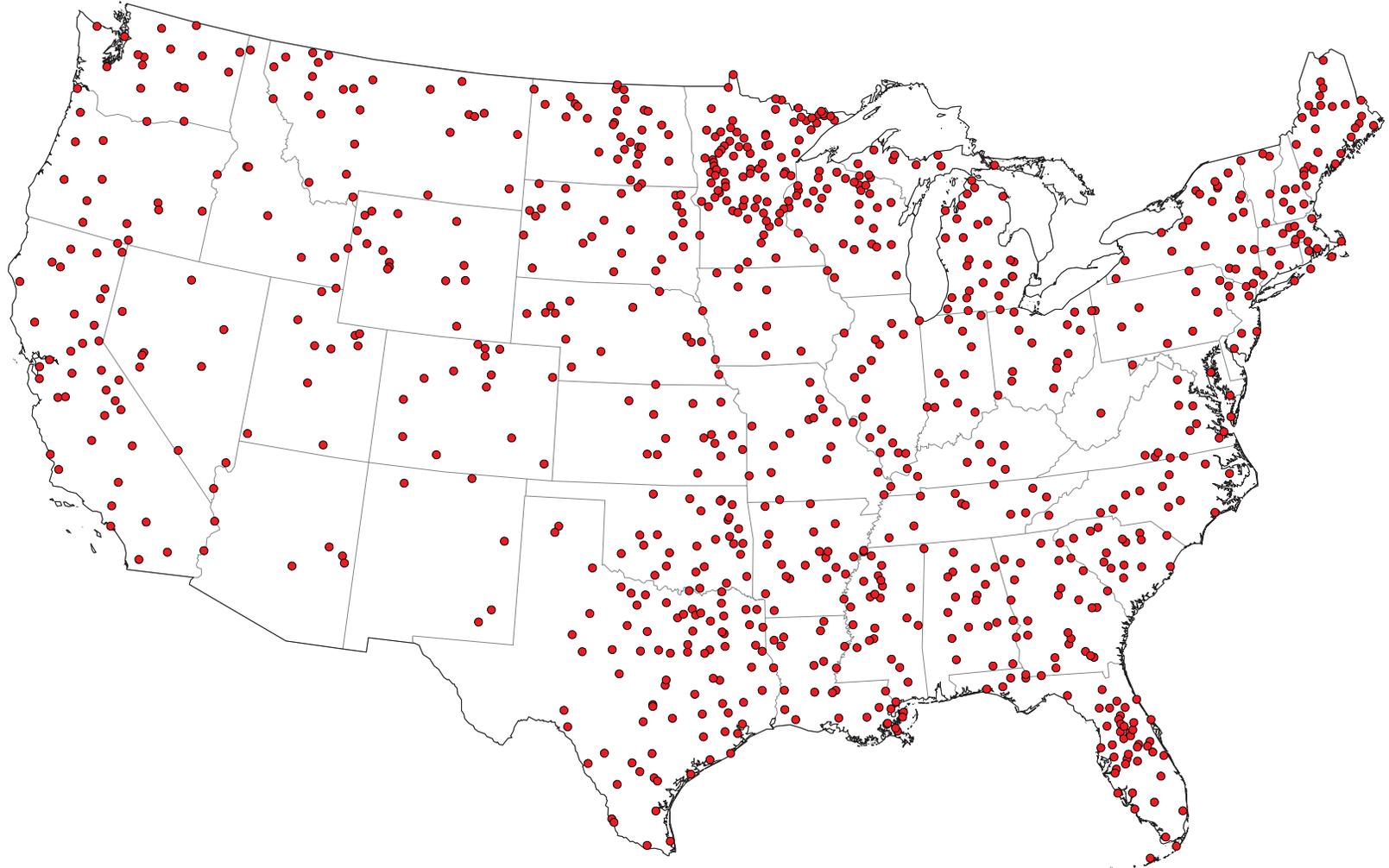


## *RF3 Sample Frame: Lakes*

<b>Lake Area (ha)</b>	<b>Number of Lakes</b>	<b>Percent</b>	<b>Cumulative Number of Lakes</b>	<b>Cumulative Percent</b>
<b>1–5</b>	172,747	63.8	172,747	63.8
<b>5–10</b>	44,996	16.6	217,743	80.4
<b>10–50</b>	40,016	14.8	257,759	95.2
<b>50–500</b>	11,228	4.1	268,987	99.3
<b>500–5000</b>	1,500	0.6	270,387	99.9
<b>&gt;5000</b>	274	0.1	270,761	100.0



# National Fish Tissue Contaminant Lake Survey

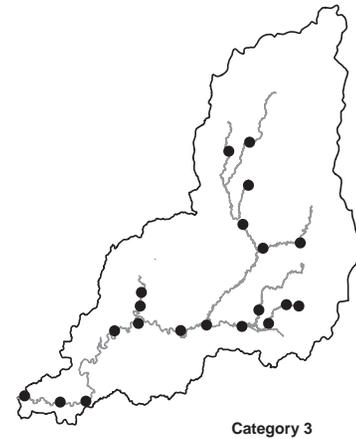
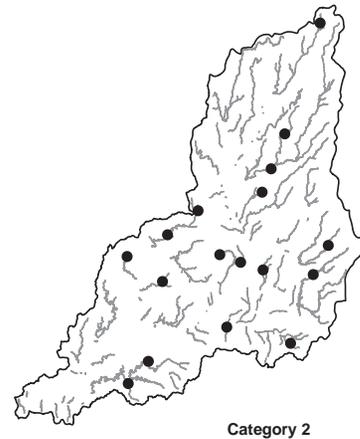
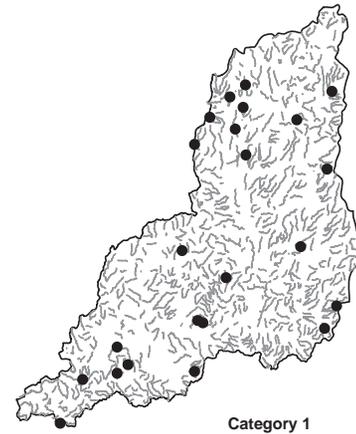
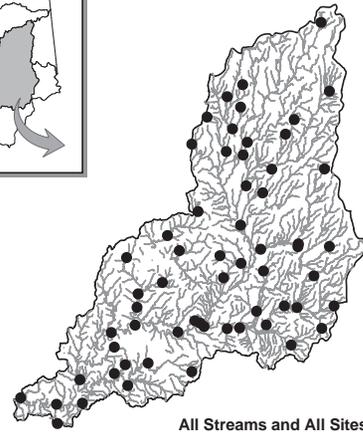


## *Sample Selected: Lakes*

<b>Lake Area (ha)</b>	<b>1999</b>	<b>2000</b>	<b>2001</b>	<b>2002</b>	<b>All Years</b>	<b>Expected Weight</b>
<b>1-5</b>	39	41	47	47	174	938.84
<b>5-10</b>	44	40	47	46	177	261.61
<b>10-50</b>	32	47	46	25	150	256.51
<b>50-500</b>	34	37	29	34	134	85.06
<b>500-5000</b>	36	30	31	41	138	11.36
<b>&gt;5000</b>	40	30	25	32	127	2.21
<b>Total</b>	225	225	225	225	900	



# *GRTS Sample of Streams*



# *GRTS Site Selection using `psurvey.design`*

- Create sample frame as shape file
  - ArcGIS, Arcview
  - Shape file must contain points, lines, or polygons
  - Make sure a “.prj” file is produced so have projection information
- Select sample: R program
  - Start R program
  - Load `psurvey.design` library (install first if not done yet)
  - Input:
    - Read dbf file in R
    - Specify the survey design
    - Select sites using “grts” command
  - Output:
    - Point shapefile of selected sites



```
# Load psurvey.design library
# Read dbf file
att <- read.dbffile('eco_13_ut')
head(att)
# specify design
Equaldsgn <- list(PanelOne=115,
                 seltype='Equal'))

Equalsites <- grts(design=Equaldsgn,
                  src.frame='shapefile',
                  in.shape='eco_13_ut',
                  att.frame=att,
                  type.frame='area',
                  DesignID='UTEco3EQ',
                  shapefile=TRUE,
                  prj='eco_13_ut',
                  out.shape='Eco3.EqualSites' )
```

## **Example R script to sample Utah landscape**



## *Comments*

- GRTS using R can be applied in one dimension
- GRTS conceptually extends to sampling 3-d or greater dimensions
- X,Y coordinates can be any continuous variables



# *GRTS Implementation Process*

- Randomly place a square over the sample frame geographic region
- Construct hierarchical grid (e.g. 'quadtree') with hierarchical addressing in the square
- Construct Peano mapping of two-space to one-space using hierarchical addressing
- Complete hierarchical randomization of Peano map
- Place sample frame elements on line in one-space using hierarchical randomization order, assigning length to element based on frame and inclusion density (unequal probability)
- Select systematic sample with random start from line
- Place sample in reverse hierarchical order

