

Using Bayesian Methods to Help Identify Impaired Waters

Gary Hess

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Characteristics of Impaired Water Lists (cont'd):

- ...
- The process for identifying impaired waters is an adjudication

Characteristics of Impaired Water Lists (cont'd):

- ...
- The process for identifying impaired waters is an adjudication
 - Public process
 - Free proof
 - Preponderance of evidence
 - Duty to explain

Relevance of Bayes' Theorem

In April 2016, Amazon search for books published from 2012 to present with “Bayesian” in the title returned 395 items

Relevance of Bayes' Theorem (cont'd)

In April 2016, a Google Scholar search for articles, since 2012, using search term “Bayesian” returns approximately 299,000 citations, including:

- 174 citations for Bayesian and “Zika”;
- 289 for Bayesian and “horse racing”;
- 1430 for Bayesian and “Ebola”;
- 1770 for Bayesian and “space exploration”;
- 3570 for Bayesian and “adjudication”;
- 4390 for Bayesian and “paleontology”;
- 4570 for Bayesian and “football”;
- 11,800 for Bayesian and “forensics”;
- 14,200 for Bayesian and “fisheries”;
- 14,900 for Bayesian and “stock market”;
- 15,700 for Bayesian and “geology”;
- 16,800 for Bayesian and “pollution”;
- 17,100 for Bayesian and “risk assessment”
- 19,900 for Bayesian and “epidemiology”; and
- 26,400 for Bayesian and “physics”.

Federal Agency Actions Relying On Bayesian Analysis

National Ocean and Atmospheric Administration

- ESA listing of marine mammal as endangered , 77 FR 70915 (2012)

Fish and Wildlife Service

- ESA listing of Pacific walrus as endangered or threatened, 76 FR 7633 (2011)
- ESA determination that polar bear is threatened, 73 FR 28212 (2008)
- Mourning Dove Harvest Strategy, to set hunting regulations (2015)

Animal and Plant Health Inspection Service

- Rule addressing risk from “Mad Cow” disease, 72 FR 53314, 53318-24 (2007)

Occupational Safety and Health Administration

- Rule addressing occupational exposure to methylene chloride, 62 FR 1494 (2007)

Coast Guard

- Search and Rescue Optimal Planning System (SAROPS)

Getting to Bayes' Theorem: A Toy Example

In 2015 the Oakland Athletics won 68 games during its 162 game season. Sonny Gray was the starting pitcher for 31 of the games played; of those 31 games, the A's won 17.

$$P(\textit{Win}) = 68/162 \approx .42$$

$$P(\textit{Gray}) = 31/162 \approx .19$$

$$P(\textit{Win} | \textit{Gray}) = 17/31 \approx .55$$

$$P(\textit{NotGray}) = 131/162 \approx .81$$

$$P(\textit{Win} | \textit{NotGray}) = (68 - 17) / (162 - 31) \approx .39$$

Getting to Bayes' Theorem: A Toy Example

$$P(\textit{Gray}|\textit{Win}) = \frac{P(\textit{Win}|\textit{Gray})P(\textit{Gray})}{P(\textit{Win}|\textit{Gray})P(\textit{Gray}) + P(\textit{Win}|\textit{NotGray})P(\textit{NotGray})}$$

Getting to Bayes' Theorem: A Toy Example

$$P(\textit{Gray}|\textit{Win}) = \frac{P(\textit{Win}|\textit{Gray})P(\textit{Gray})}{P(\textit{Win}|\textit{Gray})P(\textit{Gray}) + P(\textit{Win}|\textit{NotGray})P(\textit{NotGray})}$$

$$P(\textit{Gray}|\textit{Win}) = \frac{.55 \times .19}{(.55 \times .19) + (.39 \times .81)} = \frac{.10}{.10 + .32} = .24$$

Getting to Bayes' Theorem: A Toy Example

$$P(\text{Gray}|\text{Win}) = \frac{P(\text{Win}|\text{Gray})P(\text{Gray})}{P(\text{Win}|\text{Gray})P(\text{Gray}) + P(\text{Win}|\text{NotGray})P(\text{NotGray})}$$

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|\bar{A})P(\bar{A})}$$

Bayes' Theorem

An equation to calculate the probability of an event given evidence of a condition that relates to that event:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|\bar{A})P(\bar{A})}$$

Bayes' Theorem

A side note about likelihood, and determining whether evidence is relevant :

$$P(A|B) = \frac{P(B|A)}{P(B|A)P(A) + P(B|\bar{A})P(\bar{A})} \times P(A)$$

Candidate Techniques to Estimate a Prior Probability Distribution, and a Prediction

The Prior Probability Distribution

- Estimated from monitoring data used in prior assessments
- Estimated from monitoring data from upstream or downstream reaches
- Estimated by considering the subject waterbody to be a random sample from a population of waterbodies
- Asking the public for alternative techniques

A Prediction

Suggested Material

- **Book-length tutorials or textbooks**

Kruschke JK, Doing Bayesian Data Analysis: A Tutorial with R and BUGS (2015)

McElreath R, Statistical Rethinking: A Bayesian Course with Examples in R and (2016)

Hobbs NT, Hooten MB, Bayesian Models: A Statistical Primer for Ecologists (2015)

McCarthy MA, Bayesian Methods for Ecology (2007)

- **Chapter-length treatment in a popular book**

Silver N, The Signal and the Noise: Why So Many Predictions Fail - But Some Don't, chap. 8 (2012)

Suggested Material (cont'd)

- Articles

Austin PC, Brunner LJ, Hux JE, Bayeswatch: an overview of Bayesian statistics, *Journal of Evaluation in Clinical Practice* 8(2):277-286 (2002)

Brown BS, Detenbeck NE, Eskin R, How probability survey data can help integrate 305(b) and 303(d) monitoring and assessment of state waters, *Environmental Monitoring and Assessment* 103(1-3):41-57 (2005)

Duan Y, Smith EP, Ye KY, Using power priors to improve the binomial test of water quality, *Journal of Agricultural, Biological, and Environmental Statistics* 11(2):151-168 (2006)

Ellison AM, An introduction to Bayesian inference for ecological research and environmental decision-making, *Ecological Applications* 6(4):1036-46 (1996)

Fienberg SE, Bayesian Models and Methods in Public Policy and Government Settings, *Statistical Science* 26(2):212-226 (2011)

Fienberg SE, Schervish MJ, The relevance of Bayesian inference for the presentation of statistical evidence and for legal decisionmaking, *Boston University Law Review* 88:771-798 (1986)

Germano JD, Ecology, statistics, and the art of misdiagnosis: the need for a paradigm shift, *Environmental Reviews* 7(4): 167-190 (1999)

Gerrodette T, Inference without significance: measuring support for hypotheses rather than rejecting them, *Marine Ecology - An Evolutionary Perspective* 32(3):404-418 (2011)

Suggested Material (cont'd)

- Articles (cont'd)

Greenland S, Bayesian perspectives for epidemiological research: I. Foundations and basic methods, *International Journal of Epidemiology* 35(3):765-775 (2006)

Hobbs NT, Hilborn R, Alternatives to statistical hypothesis testing in ecology: A guide to self teaching, *Ecological Applications* 16(1):5-19 (2006)

Kuhnert PM, Martin TG, Griffiths SP, A guide to eliciting and using expert knowledge in Bayesian ecological models, *Ecology Letters* 13(7):900-914 (2010)

LoBuglio JN, Characklis GW, Serre ML, Cost-effective water quality assessment through the integration of monitoring data and modeling results, *Water Resources Research* 43(3):W03435 (2007)

Low Choy S, O'Leary R, Mengersen K, Elicitation by design in ecology: using expert opinion to inform priors for Bayesian statistical models, *Ecology* 90(1):265-277 (2009)

McCarthy MA, Masters P, Profiting from prior information in Bayesian analyses of ecological data, *Journal of Applied Ecology* 42(6):1012-1019 (2005)

Pascual P, Wagner W, Fisher E, Making method visible: improving the quality of science-based regulation, *Michigan Journal of Environmental and Administrative Law* 2:429-471 (2013)

Runge MC, Converse SJ, Lyons JE, Which uncertainty? Using expert elicitation and expected value of information to design an adaptive program, *Biological Conservation* 144(4):1214-1223 (2011)

Ye K, Smith EP, A Bayesian approach to evaluating site impairment, *Environmental and Ecological Statistics* 9(4):379-392 (2002)

Suggested Material

- Law reviews and related

Aitken CGG, Taroni F, Statistics and the Evaluation of Evidence for Forensic Scientists (2004)

Bressman LS, Procedures as politics in administrative law, Columbia Law Review 107:1749-1821 (2007)

Charest S, Bayesian approaches to the Precautionary Principle, Duke Environmental Law and Policy Forum 12:265 (2002)

Nance DA, Naturalized epistemology and the critique of evidence theory, Virginia Law Review 87:1551 (2001)

Pascual P, Wagner W, Fisher E, Making method visible: improving the quality of science-based regulation, Michigan Journal of Environmental and Administrative Law 2:429-471 (2013)

- Online resources

O'Hagan A, Luce BR, A Primer on Bayesian Statistics in Health Economics and Outcomes Research (2003) at https://www.shef.ac.uk/polopoly_fs/1.80635!/file/primer.pdf

Etz A, How to Become a Bayesian in Eight Easy Steps (2016), at <https://alexanderetz.com/2016/02/07/understanding-bayes-how-to-become-a-bayesian-in-eight-easy-steps/>