Estimation of Nonlinear Trends Using High-frequency Water-quality Monitoring Data

Guoxiang Yang*, Douglas Moyer

U.S. Geological Survey, Richmond, VA

*Natural Systems Analysts

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Motivation

- ❖ Increasing collection of high-frequency water data
- ❖ A common question: is water quality improving over time
- ❖ Traditional trend estimation method
- ❖ Trend estimation method for high frequency data

Motivation

- Start with seasonal-trend decomposition procedure
  - Seasonal, trend and the remainder components
  - Atmospheric CO_2 concentration and temperature
- Need a universal approach to model different constituents
  - Ranging from water temperature to turbidity
- Generalized Additive Models (GAMs)

Study Area

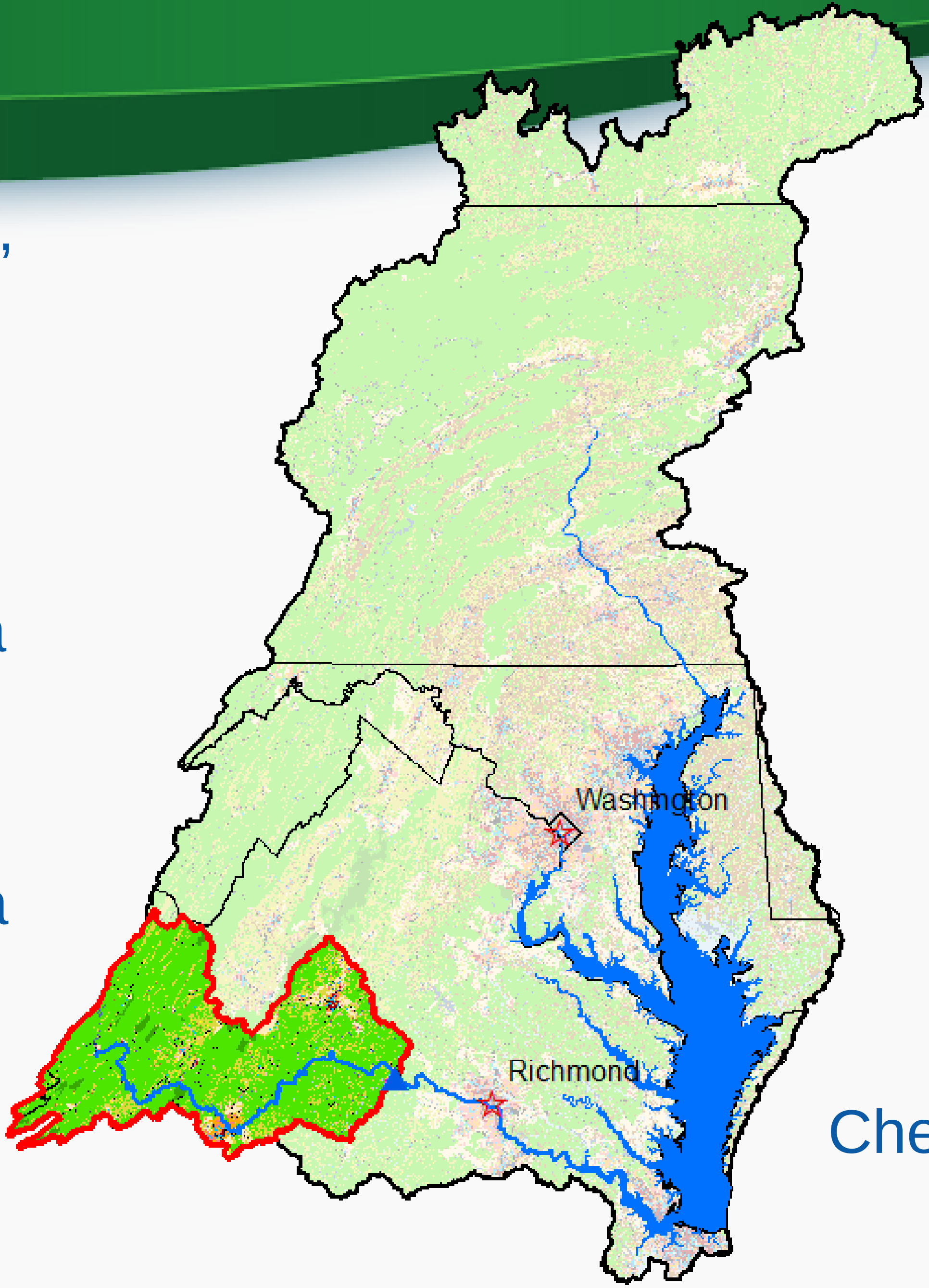
James River at Cartersville, VA (02035000)

Area: 16,190 km^2

The longest river in Virginia

15-min high frequency data

Mean daily value for trend estimation



2012 land use map

Urban: 7.5%

Ag: 16%

Forest: 73%

Chesapeake Bay

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Generalized Additive Model (GAM)

Generalized: many response distributions other than normal

Additive: terms add together

$$g\left(E(y_i)\right) = \beta_0 + f_1(x_{1i}) + f_2(x_{2i}) + \ldots + \varepsilon_i \qquad (1)$$

g is a link function

$y_i \sim$ some exponential family distribution

f_1, f_2 … are unknown smooth functions

x_1, x_2 … are covariates

ε_i can have random effects

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GAM Theory - Smooth Function

Smooth function (spline): sum of basis functions *b*

and their corresponding regression coefficients β

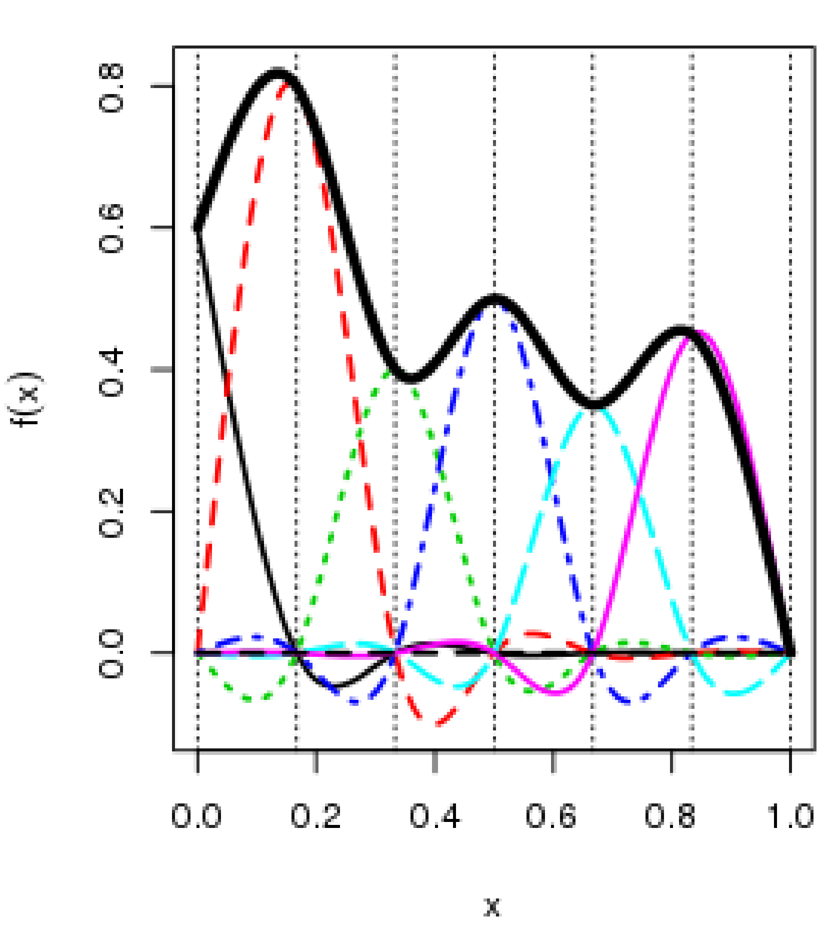
$$f(x) = \sum_{j=1}^{k} b_j(x)\beta_j$$

k: number of knots - basis dimensions used for the spline

The model (1) can be written in a linear way:

$$g\big(E(y)\big) = X\beta + \varepsilon \qquad (2)$$

GAM - a generalized linear model (GLM) with linear predictor involving a sum of smooth functions of covariates, subject to smoothing penalties



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GAM Theory – Correlation in Data

- GAMs assume that residuals are identically and independently distributed (i.i.d.)

  This assumption is too strong for daily time series

- Time-series data: sequential time points will be highly correlated
- Include autoregressive moving average (ARMA) model for errors in GAM model
- Generalized Additive Mixed Model (GAMM): to handle residual correlation

$$g\big(E(y)\big) = X\beta + \varepsilon \qquad (2)$$

$$g\big(E(y)\big) = X\beta + A + \varepsilon \qquad (3)$$

Water Temperature GAM Model

Water Temperature ~ s(season) + s(time)

```
Formula:
WT ~ s(nDay, bs = "cc", k = 20) + s(Time, bs = "cr", k = 20)

Parametric coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  16.2111     0.1055   153.7   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Approximate significance of smooth terms:
           edf Ref.df       F p-value
s(nDay) 13.169 18.000 367.694 < 2e-16 ***
s(Time)  7.378  7.378   3.871 0.00025 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

R-sq.(adj) =   0.94
  Scale est. = 5.1415    n = 4401
```

s(nDay) is the smoothing function of seasonal covariate.

s(time) is the smoothing function of time covariate.

"cr": cubic spline basis

"cc": cyclic cubic spline

Adj-R^2

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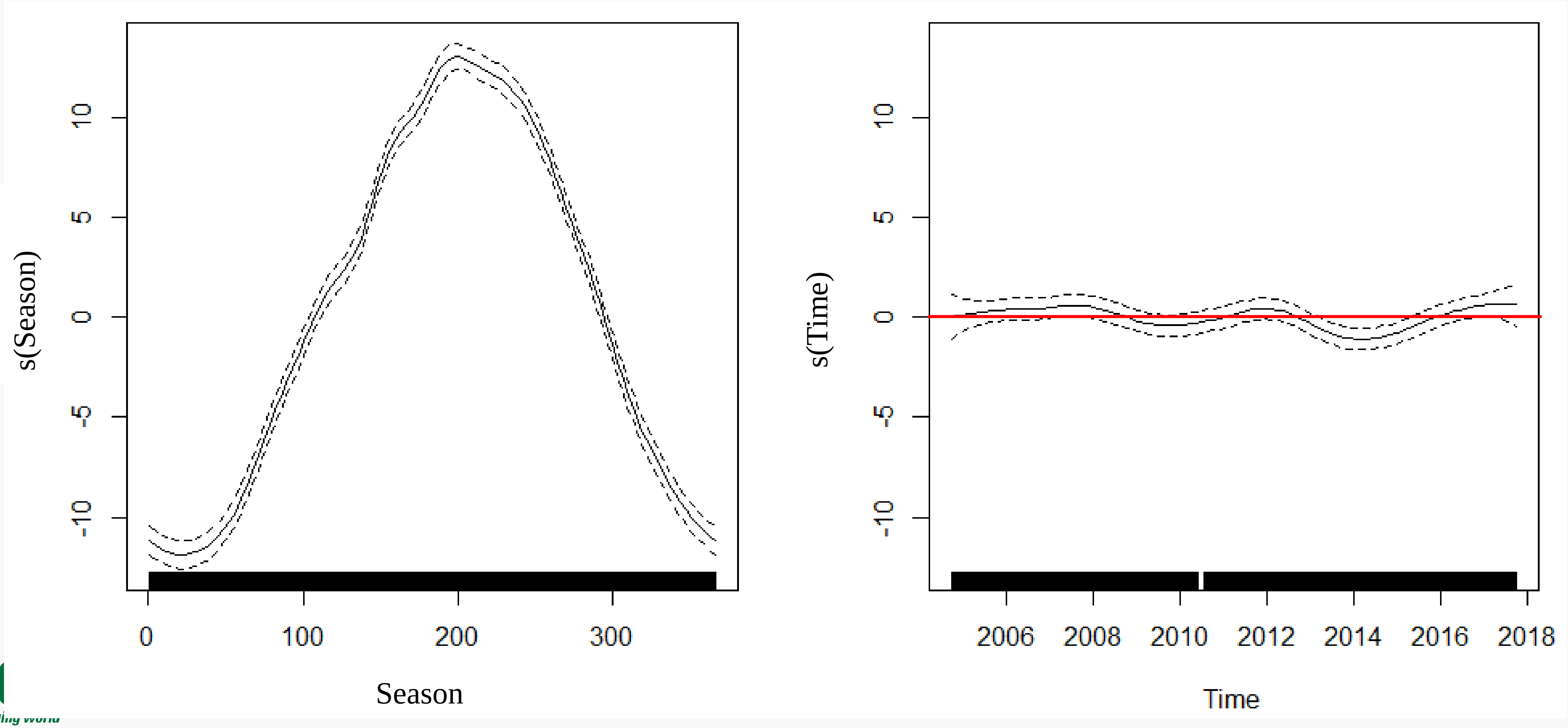
Water Temperature GAM Model

- Water Temperature ~ s(season) + s(time)
- The effect of each smooth function on water temperature



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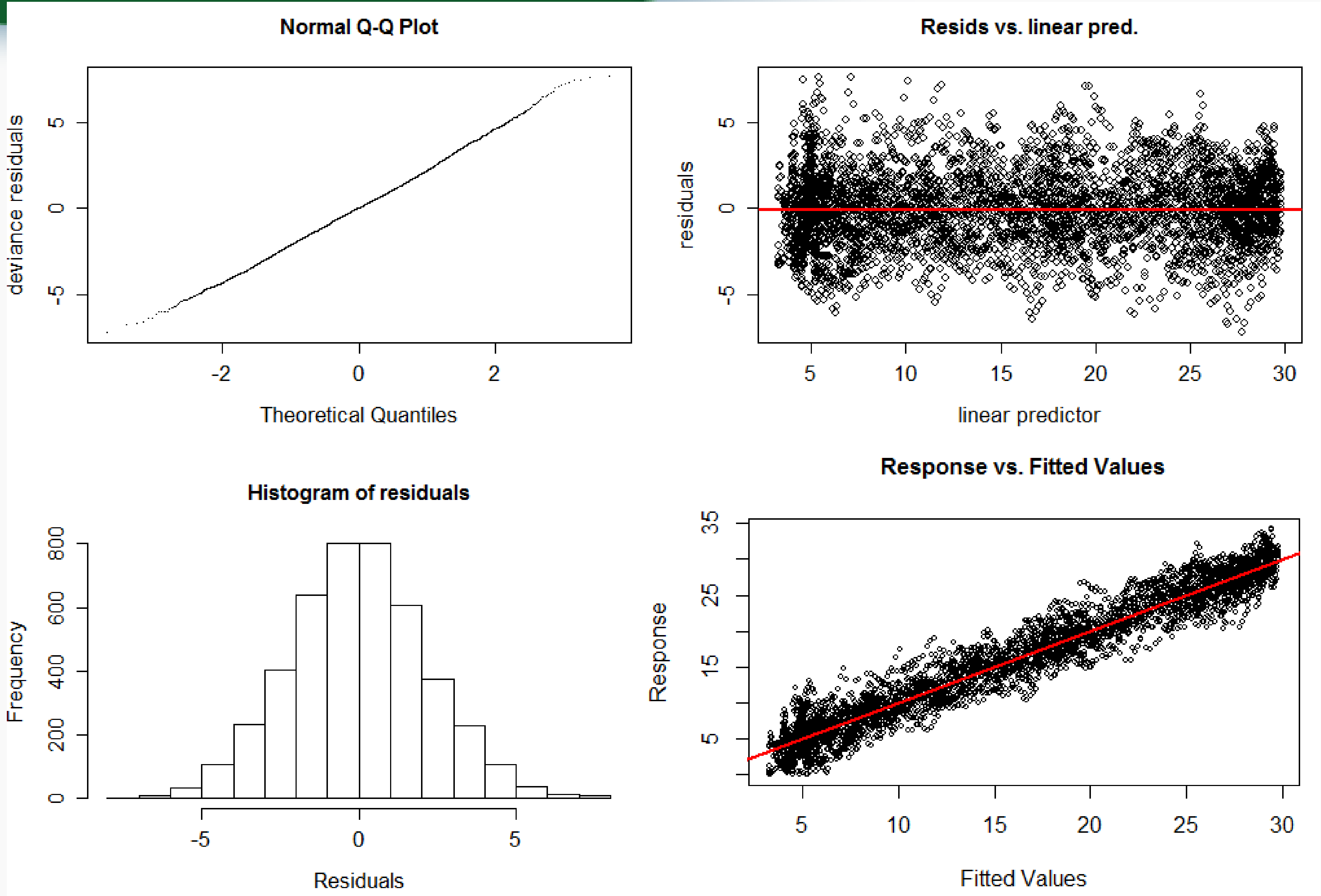
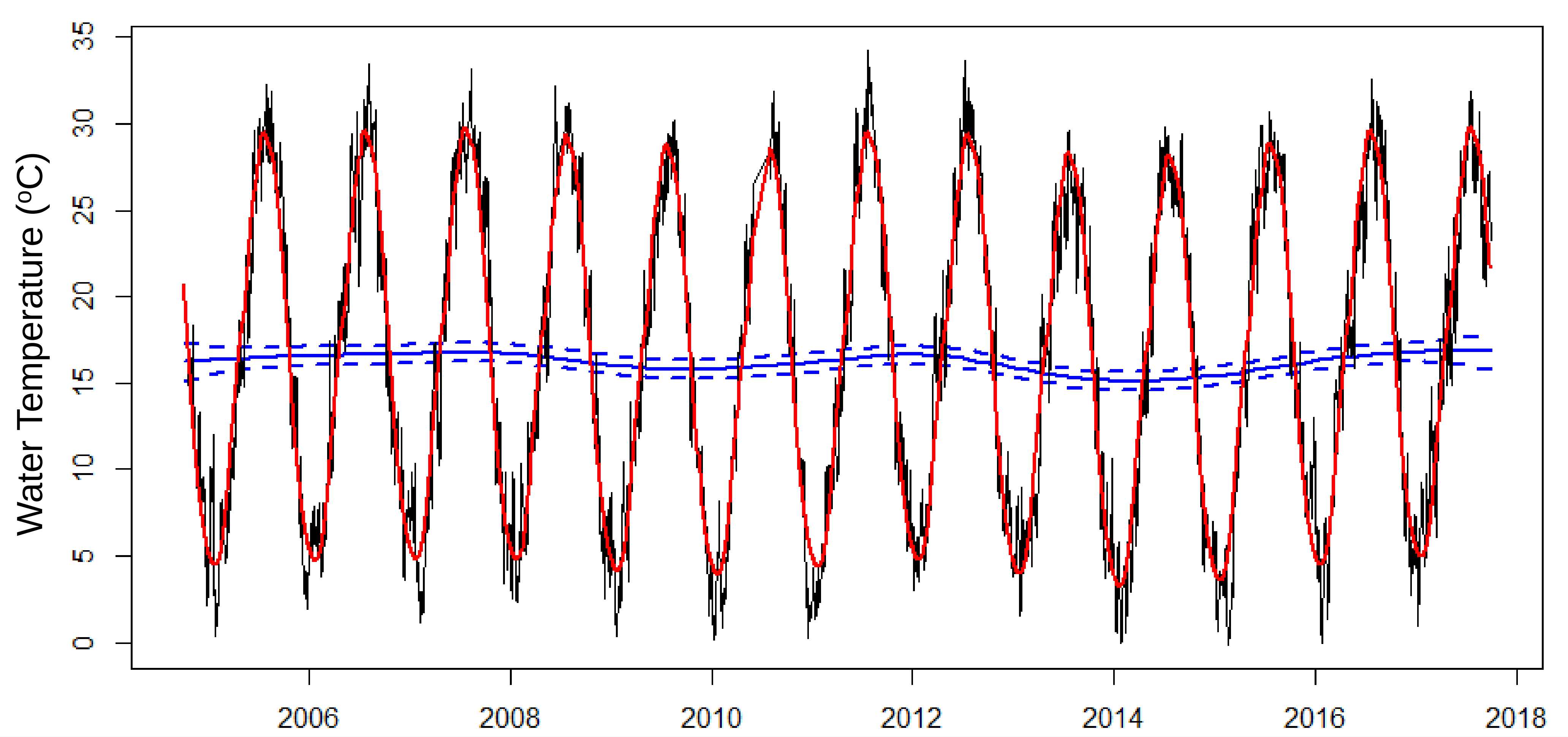
Water Temperature GAM Model



Water Temperature – Time Series



Obs Sim s(Time) Mean = 16.21 °C

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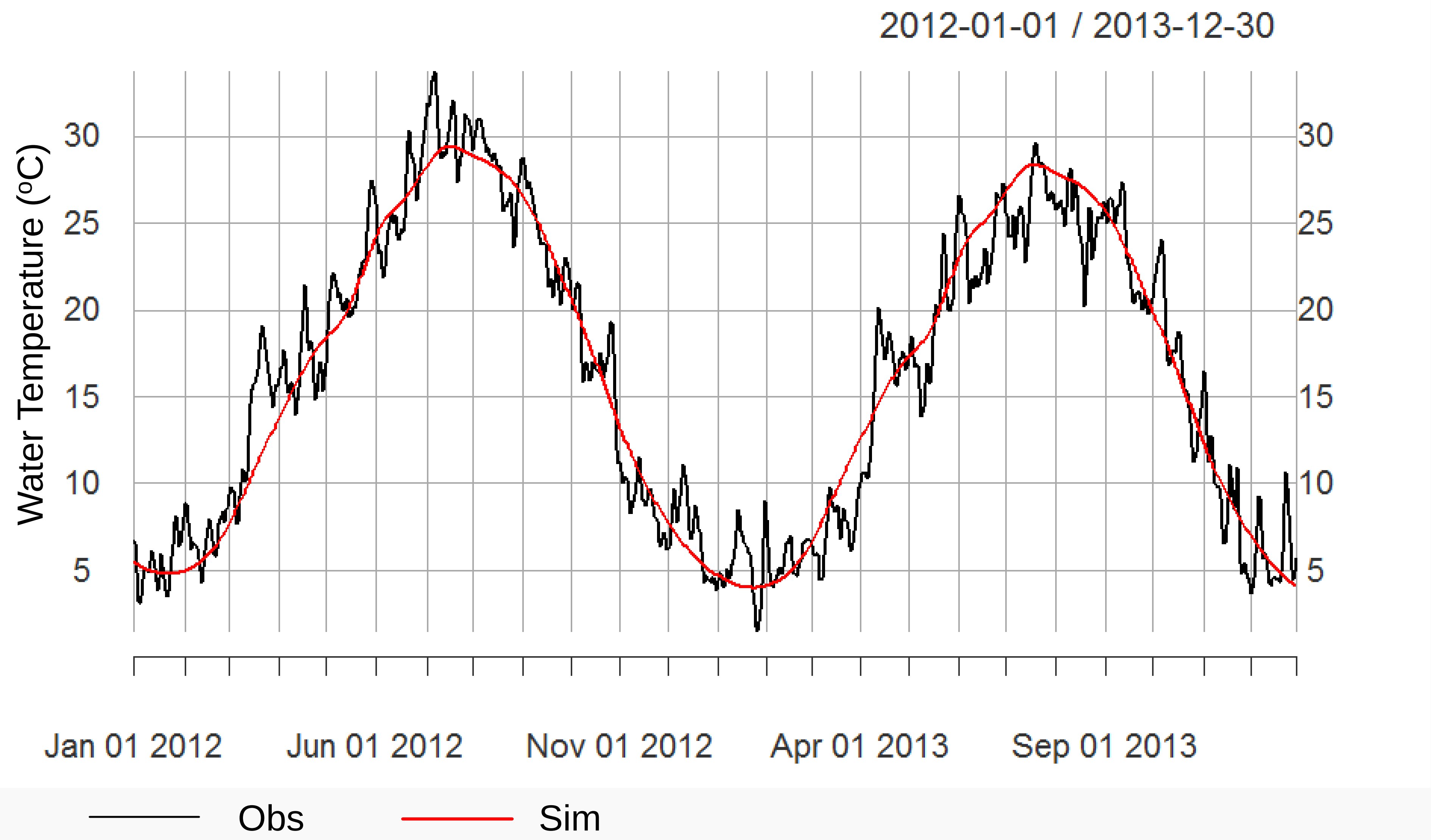
Water Temperature – Time Series



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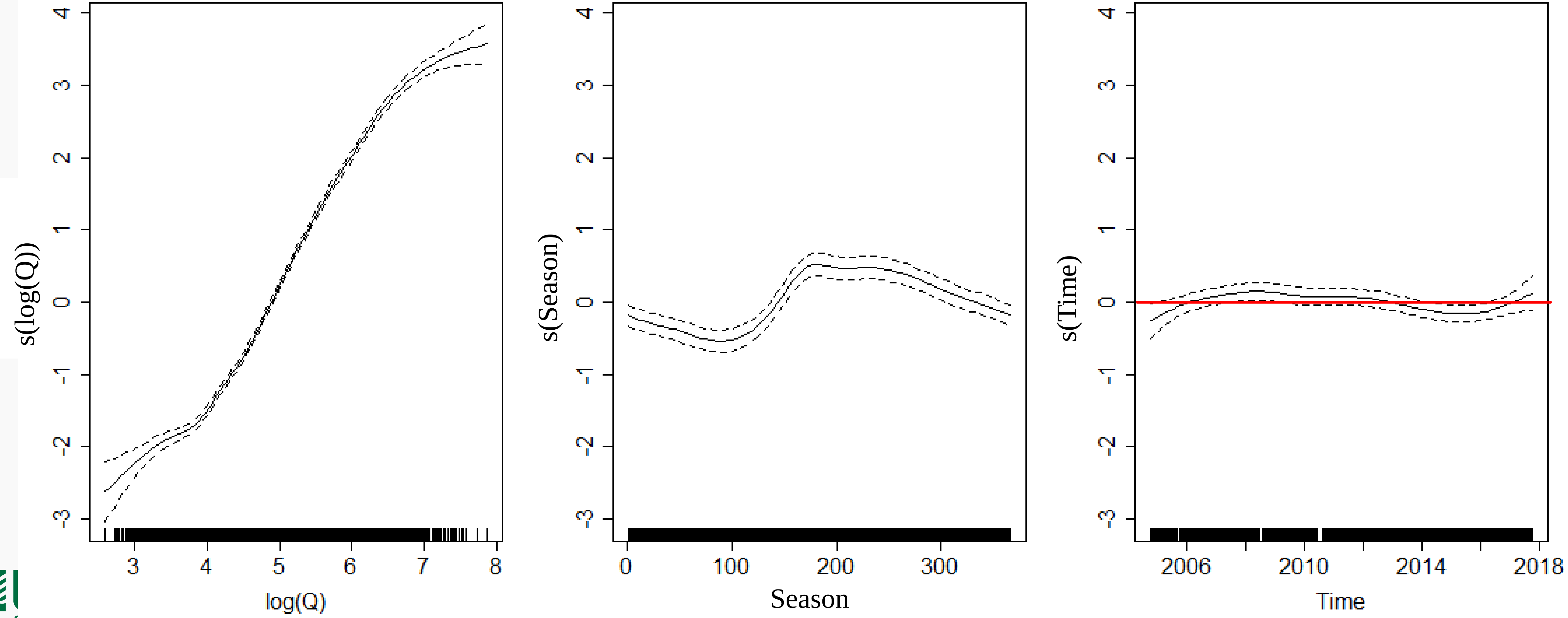
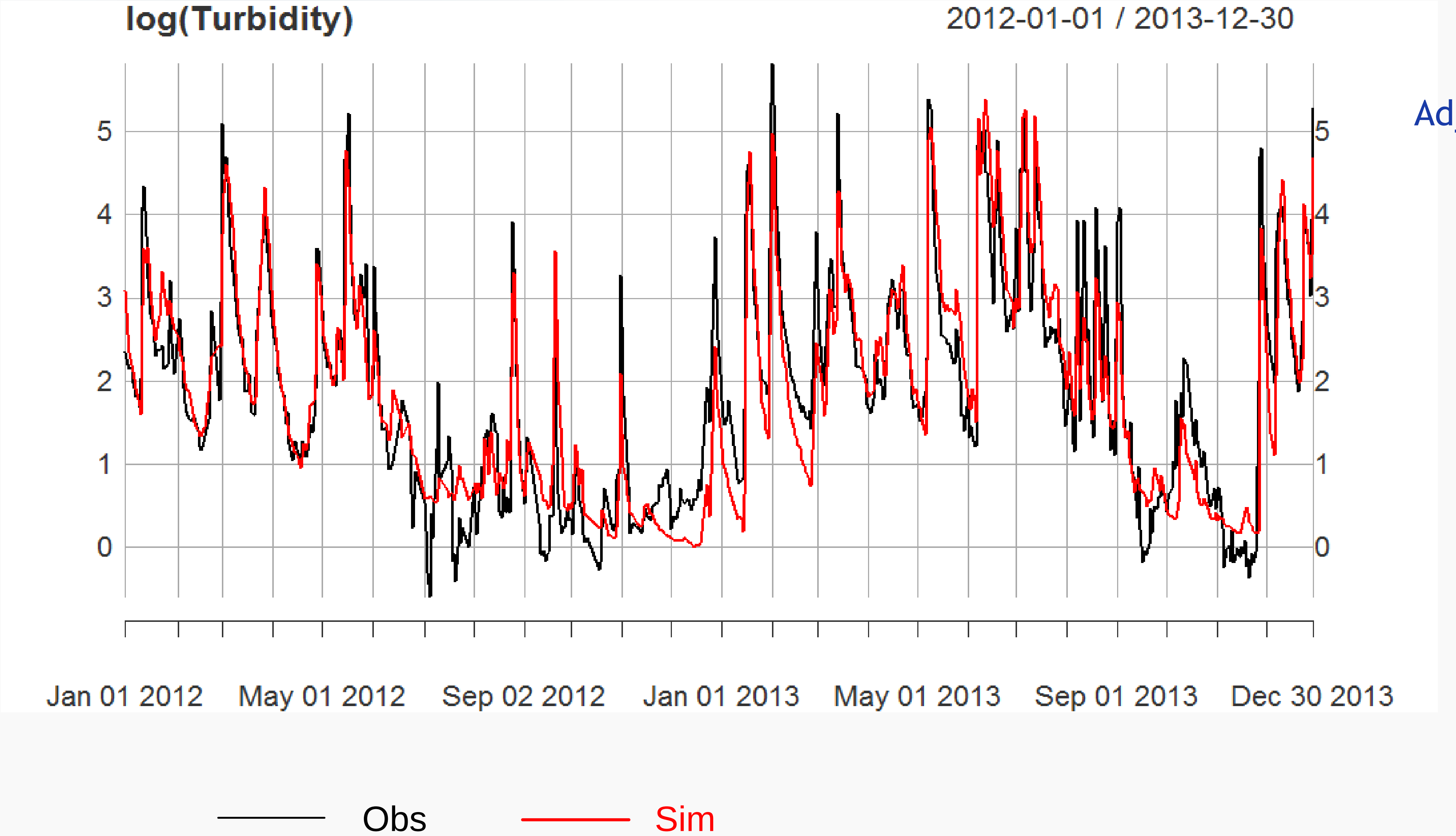
Turbidity GAM Model

- Log(Turbidity) ~ s(log(Q)) + s(season) + s(time)
- The effect of each smooth function on turbidity



Log-Turbidity - Simulation vs. Observation



Adj-R^2 = 0.79

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Trend & CI Estimation Between Two End-points

- GAM model has an underlying parametric representation

$$\hat{\eta}_p = f(time) = X_p\hat{\beta}$$

$\hat{\eta}_p$: smooth function of time component

X_p, β are model matrix and coefficient vector

- Derive the *mean* of the difference between the two endpoints (*trend*) and the standard error of this difference
- Conduct statistical inference using standard statistical theory
- Confidence interval and p-value

Water Temperature Trend Estimation

- Time period: 2007-01-01 ~ 2014-12-31
  - Trend over time period: -1.27°C

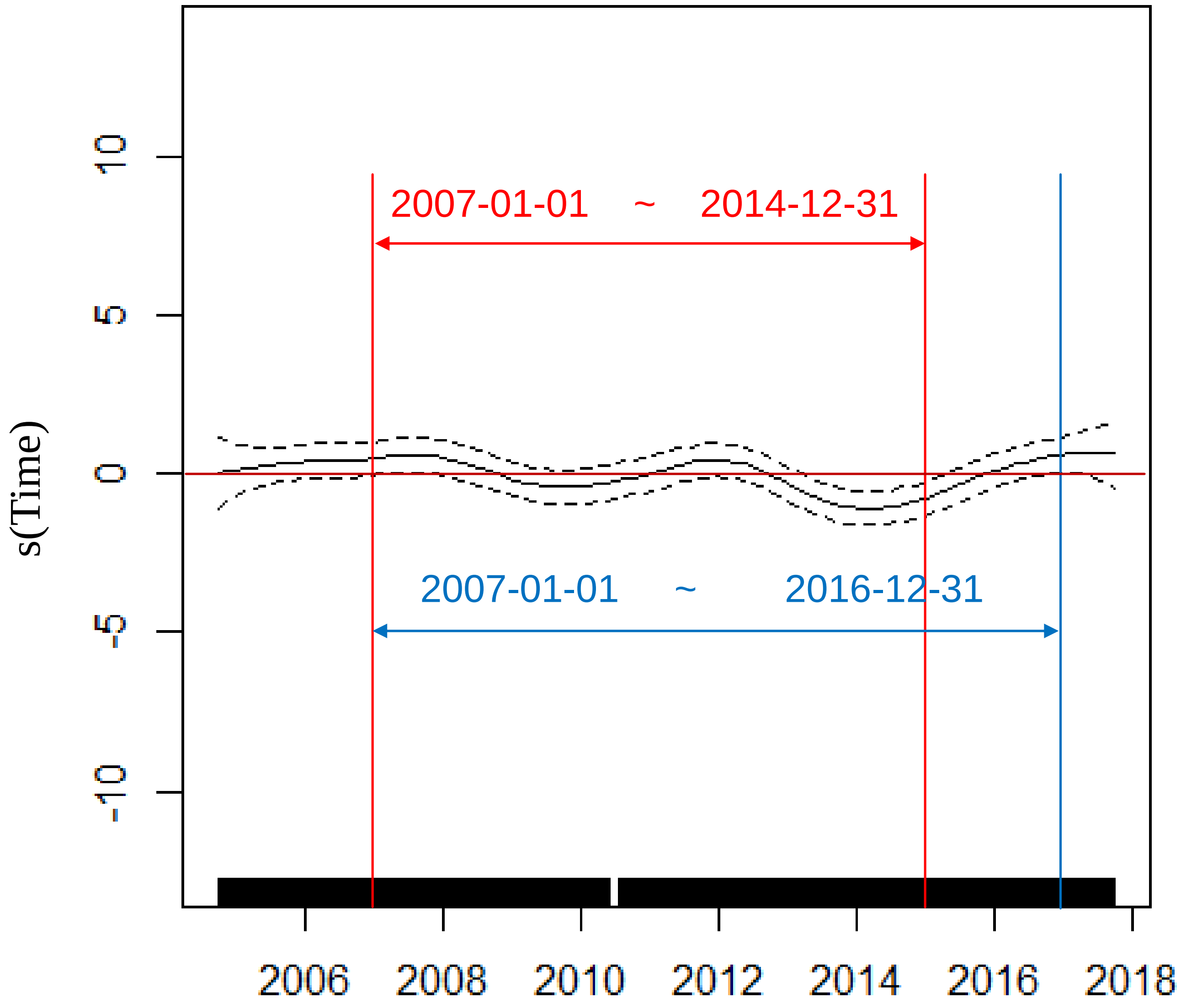
    95% confidence interval: -2.06 ~ -0.48

    p-value: < 0.01

- Time period: 2007-01-01 ~ 2016-12-31
  - Trend over time period: 0.12°C

    95% confidence interval: -0.7 ~ 0.93

    p-value: 0.78



Turbidity Trend Estimation

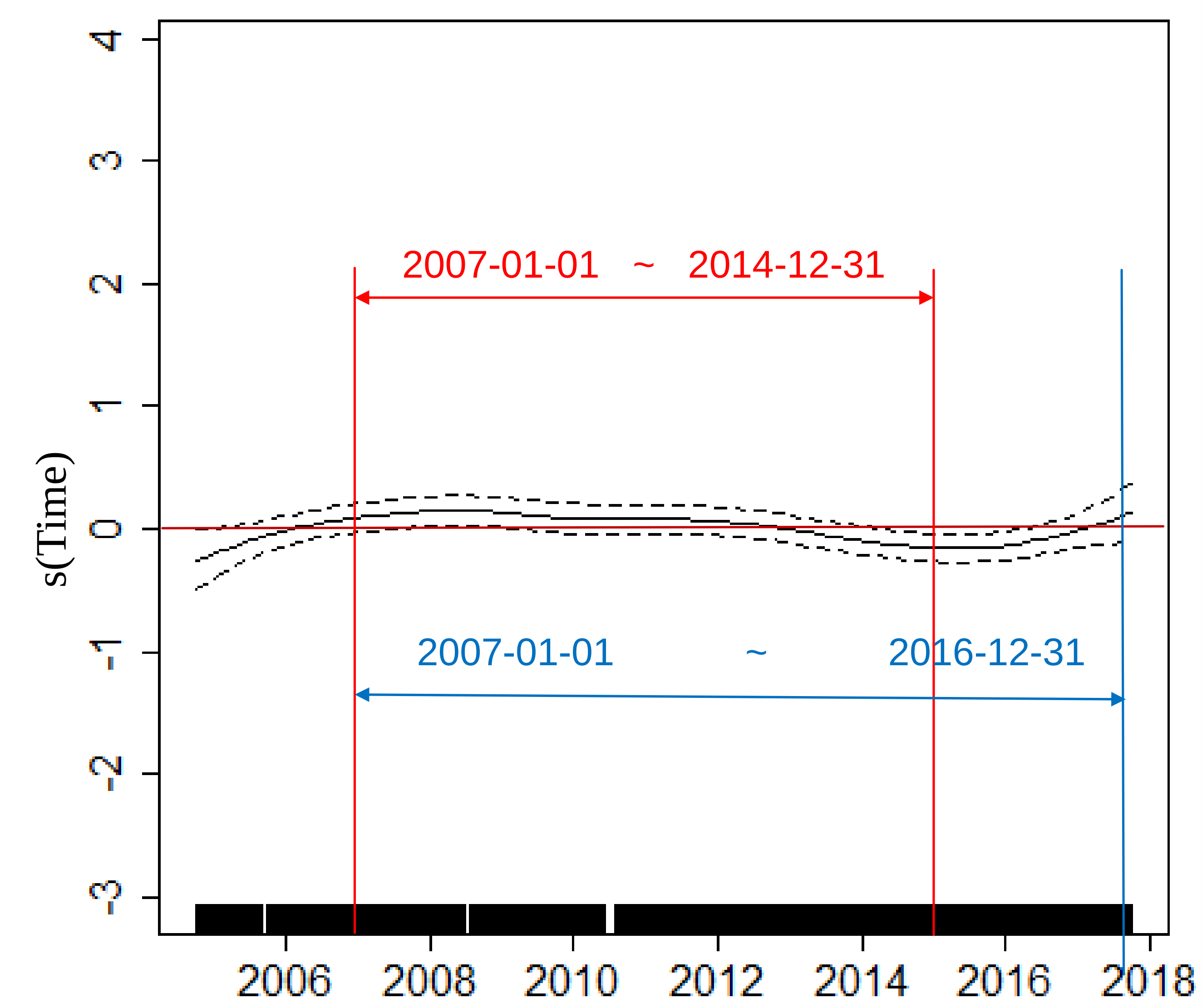
- Time period: 2007-01-01 ~ 2014-12-31
  - Trend (log scale): -0.24 FNU ($p < 0.01$)

    95% confidence interval: -0.42 ~ -0.06

- Time period: 2007-01-01 ~ 2016-12-31
  - Trend (log scale): -0.10 FNU ($p = 0.30$)

    95% confidence interval: -0.30 ~ 0.09

- Interpretation of the trend signal between estimation endpoints
  - How can we utilize the variability in the trend time series to better understand/link water-quality responses to episodic changes in the watershed (e.g. stream restoration, floods, or urbanization)?



GAM Trend Estimation Summary

- It is flexible to incorporate many possible predictors
- Decompose the signal into different components over time
- What does the trend pattern looks like over time
- Test if the trend is significant over a given time period
- Statistical inference, confidence interval calculation
- Can short-term trend signals be linked to restoration/disturbance events in the watershed?

Thank you!

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Gavin Yang

gyang@usgs.gov

Water-quality monitors deployed on the James River at Cartersville, VA