SUBSURFACE EXIT GRADIENTS AT A DRAINAGE DITCH WITH LOWER WATER TABLE THAN THE GROUNDWATER TABLE

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Abstract Rill and gully development are usually thought of as a consequence of overland flow in which at selected locations in the landscape the shear stress has exceeded a threshold value. In those cases, the flow regime causes local scour that might lead to headcuts and incisions in the concentrated flow pathways. In most of these studies, the role of the subsurface water regime as a contributing factor is being ignored. Field and laboratory studies have shown that in addition to surface flow, the subsurface soil water regime may appreciably affect conditions for incipient rilling and ephemeral gully development. Also, in places where the groundwater table intersects the surface topography, seepage occurs that leads to localized soil erosion which manifests itself in terms of incisions and headcuts. This article summarizes an analysis of the subsurface flow regime in a homogeneous isotropic aquifer with a water table that is higher than the water level in the incised drainage ditch. The aquifer with a finite depth has an open surface boundary and is infinite wide. To facilitate the analysis, a steady state flow condition is considered in which the analysis is based on a solution of Laplace's equation for the flow region. Expressions for the exit gradients at the water/soil inter-phase were determined. The model considers a buffer strip where no water enters the soil profile. Estimates are made for seepage losses for different buffer strip widths. The analysis yields a close-form solution for this simplified physical realization that will facilitate estimations of subsurface flow in runoff and erosion models.

INTRODUCTION

Soil erosion on upland areas is a highly complicated phenomenon involving many component processes. Traditionally, the main focus of soil erosion research has been on rainfall and runoff induced soil detachment and transport processes, which in landscape settings are considered to take place in rills and interrills (Foster and Meyer, 1975; Meyer et al., 1975). While these surface processes still have the interest of the research community and are embedded in important erosion prediction models such as RUSLE, AGNPS, and WEPP, recent emphasis of erosion research on upland area has shifted to ephemeral gully erosion and permanent gully development. Gully development is usually thought of as a consequence of concentrated flow that have exceeded critical or threshold values relative to volume and slope steepness or energy gradients. It is generally recognized, that gullies, and in particularly ephemeral gullies, are the most important sources of sediment in stream systems of agricultural watersheds. Ephemeral gullies as opposed to rills are located in the depression of sloping land where runoff concentrates. Once obliterated by farm implements, they return at the same location in a subsequent storm that produces runoff. During a storm event, large amounts of soil, especially loose soil, is picked up by runoff and moved downslope into these depressions, from where it subsequently is moved into the watershed stream system. This process is primarily a surface process in which, often, characteristic headcuts develop. Headcut erosion has been discussed by Bennett et al. (2000) from the hydrodynamics perspective of the surface flow and by Prasad and Römkens (2005) from the rate controlling aspect by crusts and seals of the surface profile layer. There is increasing recognition that the role of subsurface flow may play a significant role as well (Römkens et al., 1997) through the increase in soil water pressures and/or seepage that adversely affects soil stability and detachment of soil particles. Recently, the results of an analytical study was published that allowed the estimation of seepage and the evaluation of pressure potentials near an incised ditch in a homogeneous aquifer of finite thickness (Römkens, 2009). To evaluate the exit gradients at the drain or ditch boundary, two approaches are possible. The first, the approximate approach, consists of the determination of pressure hydraulic potential difference on selected streamlines between the ditch boundary and an aequipotential a small but finite distance into the aquifer. The computed coordinates of these points allow, then, a close estimate of the hydraulic gradient on the stream lines near the flow exit points. The second, the exact approach, consists of deriving an analytical expression of the hydraulic gradient at the ditch boundary itself. This article discusses the latter approach of arriving at the pressure gradient relationship at the point of water entry along the ditch surface into the stream system.

APPROACH

The model chosen consists of an incised ditch into a flat landscape with a constant, horizontal groundwater table that is higher than the water level in the ditch. The soil conducting water has a finite depth and is homogeneous and isotropic and has, therefore, a constant saturated hydraulic conductivity that is not dependent on the flow direction. The field adjacent to the ditch is ponded except for a small strip of width c along the ditch. No water is allowed to enter the soil profile through the surface of this strip. Thus, water flows through the permeable soil under a steady state regime from the field to the ditch. A physical realization of this flow region is shown in Fig. 1. The incised ditch has a circularly shaped bottom which is filled with water that is

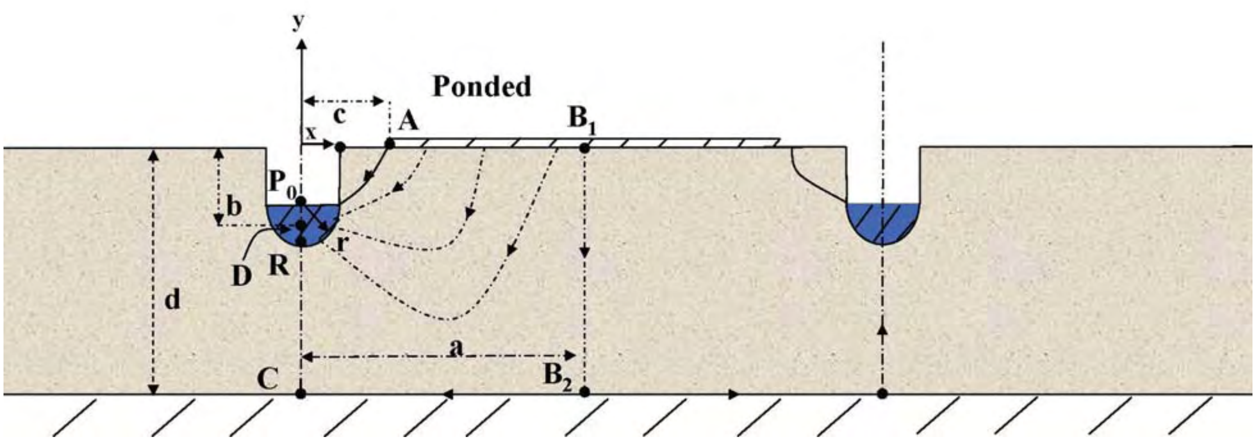

Figure 1 A schematic representation of the flow region.

maintained at a constant water level. This flow regime is, in fact, a potential flow problem that can be described by Laplace's equation in terms of potential functions φ and stream functions ψ. To facilitate the analysis, the flow region is further simplified by extending the region away from the ditch to infinity ($a \rightarrow \infty$). This flowmodel will, from now on, be called the ditch model. A second simplification can be made by shrinking the width of strip c so that in the limit c=0 and the ditch approaches that of a fully filled circular drain buried in the aquifer with a pressure in the drain equal to the difference in the water level between the water level at the soil surface and that of the ditch. This model will be called the drain model. For the drain model, the flow region is rectangular in shape. Given this simplified geometry, an analytical solution of Laplace's equation for this region can readily be obtained using a series of conformal transformations. Figure 2 shows the sequence of transformations of the complex plane z (z=x+iy) and the complex flow plane ω (ω= φ+iψ) onto the lower half of the complex plane epsilon such that the transformed vertices A (0,0), B(∞,0 and ∞,id), C (o,id), and D (0,ib) of the z-plane coincide with the transformed vertices A (0,Q_1), B (0,0), C (- φ(C),0), and D (-∞,Q_1 and -∞,0) of the ω-plane,

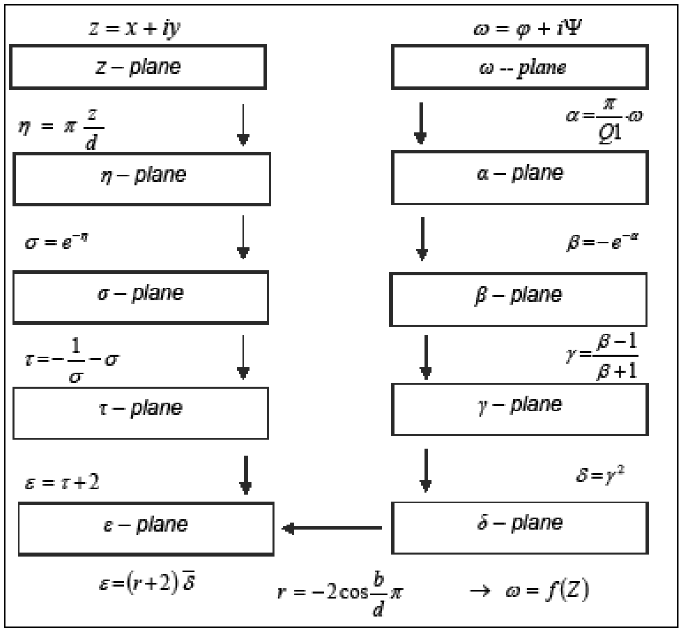

Figure 2 A schematic representation of sequential conformal transformations of the complex z-plane and ω-plane onto the upper half of the complex ε-plane for the flow from a horizontal water table to a fully filled drain.

respectively. Q_1 is the flow rate or seepage rate per unit ditch length. In that case, mutual substitutions of the transforming relationships yield the following expression for the drain model:

$$\boldsymbol{\omega} = -\frac{\boldsymbol{Q}_1}{\boldsymbol{\pi}} \ln \left[\frac{\sqrt{(1+\boldsymbol{r}/2)} + \sqrt{1 - \cosh \boldsymbol{z}\frac{\boldsymbol{\pi}}{\boldsymbol{d}}}}{-\sqrt{(1+\boldsymbol{r}/2)} + \sqrt{1 - \cosh \boldsymbol{z}\frac{\boldsymbol{\pi}}{\boldsymbol{d}}}} \right] \tag{1}$$

where r = -2 cos (b/d). The latter parameter represents the relationship between the location of the drain relative to the depth of the impermeable layer. A similar procedure can be followed for the ditch model where c has a finite width. The corresponding ditch model relationship is:

$$\omega = -\frac{Q_1}{\pi}\, ln \left[\frac{-\sqrt{cosh\frac{c}{d}\pi - cosh\frac{z}{d}\pi} - \sqrt{cosh\frac{c}{d}\pi - cos\,\frac{b}{d}\pi}}{-\sqrt{cosh\frac{c}{d}\pi - - cosh\frac{z}{d}\pi} + \sqrt{cosh\frac{c}{d}\pi - cos\frac{b}{d}\pi}} \right] \tag{2}$$

Note that Eq. (2) reduces to Eq. (1) upon substitution of c=0 (zero width of the buffer strip) which is consistent with the physical realization of the model. Three requirements must be met: 1. The hydraulic potential in the fully filled drain or on the wetted boundary of the ditch should have a constant value equal to the head difference between ditch water level and the water table on the field but adjusted for the hydraulic conductivity ($\varphi = K\Phi$); 2. The wetted circumference of the ditch model should be equal to that of the natural drainage ditch; 3. The flow lines to the ditch model should closely resemble those of the natural system. For a sink in an infinitely large 2-dimensional flow field the aequipotentials φ are concentric circles around the well center. However, because of the asymmetry of the flow field, the locations of the analytically derived aequipotentials are not concentric circles. In order to assure that the wetted boundary of the drain represents an aequipotential with a value equal to the difference between water levels of the field and ditch, the location of the well center was shifted until the location of the calculated aequipotential coincides with the perimeter of the drain or ditch.

SEEPAGE

Seepage calculations were made with the drain and ditch model. The following values were assumed in making these calculations. The difference in water tables between the field and ditch was 1.5 m, the thickness of the aquifer d=9.5 m The hydraulic conductivity was 3 m^2/day, The depth of the drain relative to the field water table was b=2m, the radius of the circularly shaped bottom of the ditch was r_0=1 m. In the calculations, three buffer strip widths were considered: c= 1, 5, and 9 m. The seepage loss as a function of buffer strip width is shown in Figure. 3. The

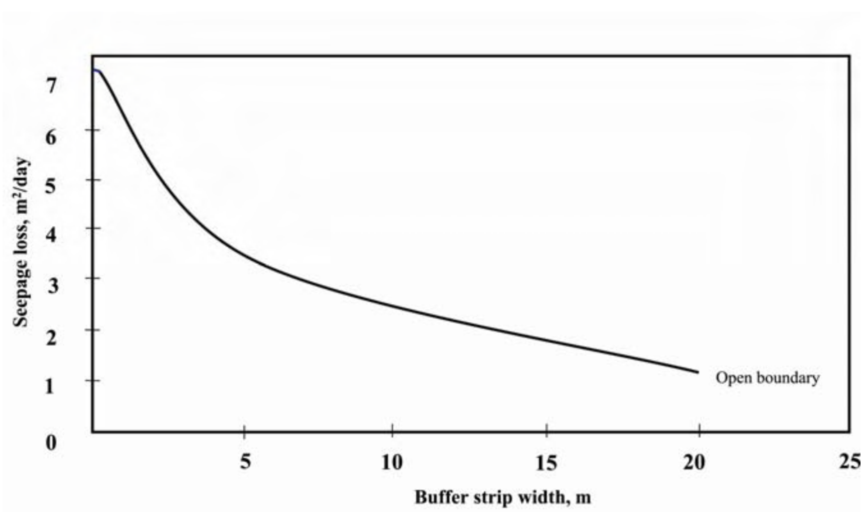

Figure 3 Seepage losses for different buffer width strip calculated for a free water boundary in the buffer strips.

accumulated water loss to the ditch as function of distance from the ditch is shown in Fig. 4 for the three buffer strips. One may notice the rapid decline of water loss from different parts of the field. Also, an increase in the width of the buffer strip shows a rapid decline in the total seepage to the ditch.

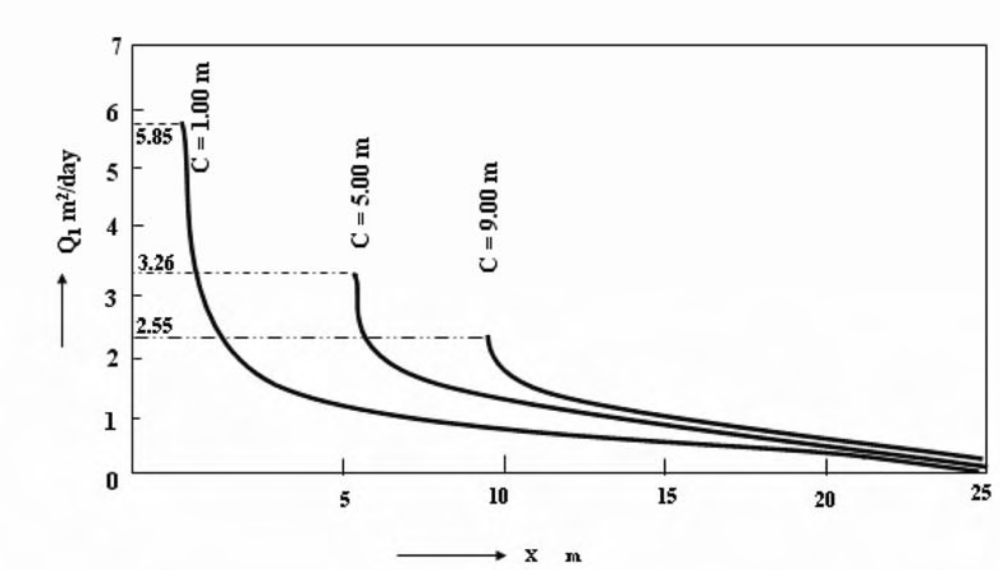

Figure 4 Accumulated seepage loss with distance from the ditch for 3 buffer zone widths.

SEEPAGE GRADIENTS

The general solution shown for this flow field in Eq. 2 indicates a close-form explicit expression with the aequipotential and streampotential functions on the right hand side (RHS) and the spatial coordinates on the left hand side (LHS). Given the explicit nature of the general solution, one can now calculate for each point z(x,y) the corresponding values of $\omega(\varphi, \psi)$. The expressions derived from Eq. 2 are:

$$\frac{\cosh\frac{c}{d}\pi - \cos\frac{y}{d}\pi\cdot\cosh\frac{x}{d}\pi}{\cosh\frac{c}{d}\pi - \cos\frac{b}{d}\pi} = \frac{e^{\frac{-4\pi\varphi}{Q_1}} - 4e^{\frac{-2\pi\varphi}{Q_1}}\cdot\sin^2\frac{\pi\psi}{Q_1} - 2e^{\frac{-2\pi\varphi}{Q_1}} + 1}{e^{\frac{-4\pi\varphi}{Q_1}} - 4e^{\frac{-3\pi\varphi}{Q_1}}\cdot\cos\frac{\pi\psi}{Q_1} + 4e^{\frac{-2\pi\varphi}{Q_1}}\cdot\cos^2\frac{\pi\psi}{Q_1} + 2e^{\frac{-2\pi\varphi}{Q_1}} - 4e^{\frac{-\pi\varphi}{Q_1}}\cdot\cos\frac{\pi\psi}{Q_1} + 1} \tag{3}$$

and

$$\frac{\sin\frac{y}{d}\pi\cdot\sinh\frac{x}{d}\pi}{\cosh\frac{c}{d}\pi - \cos\frac{b}{d}\pi} = \frac{4e^{\frac{-3\pi\varphi}{Q_1}}\cdot\sin\frac{\pi\psi}{Q_1} - 4e^{\frac{-\pi\varphi}{Q_1}}\cdot\sin\frac{\pi\psi}{Q_1}}{e^{\frac{-4\pi\varphi}{Q_1}} - 4e^{\frac{-3\pi\varphi}{Q_1}}\cdot\cos\frac{\pi\psi}{Q_1} + 4e^{\frac{-2\pi\varphi}{Q_1}}\cdot\cos^2\frac{\pi\psi}{Q_1} + 2e^{\frac{-2\pi\varphi}{Q_1}} - 4e^{\frac{-\pi\varphi}{Q_1}}\cdot\cos\frac{\pi\psi}{Q_1} + 1} \tag{4}$$

By specifying a given value for the streamline in terms of a fraction of the total seepage Q_1 in Eqs. (3) and (4), one defines in fact for each potential along the streamline the corresponding z(x,y) values. Of interest in this analysis is the seepage gradient or the potential gradient d(φ)/ds at the drain or ditch wetted boundary, where ds is the spatial differential along a given streamline. Fig. 5 shows a schematic representation of the exit gradient at the wetted boundary for a given streamline.

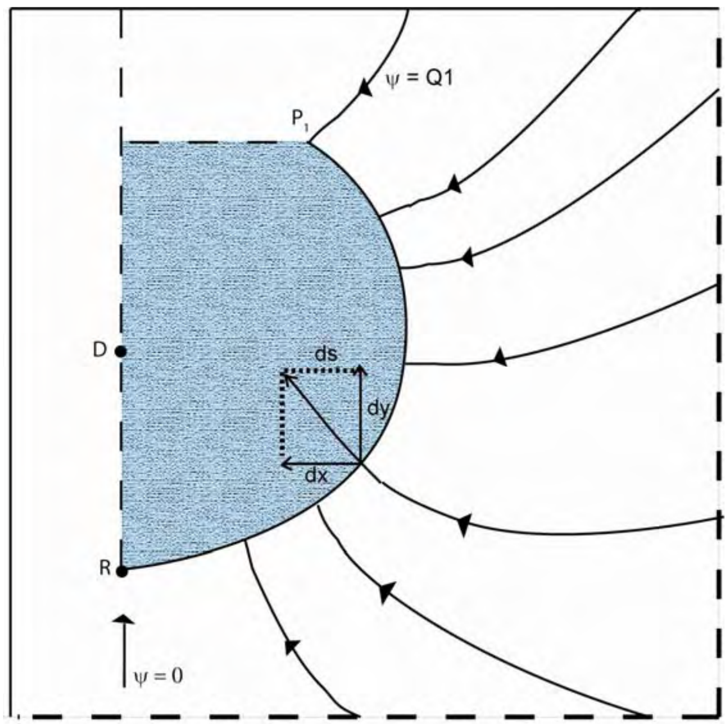

Figure 5 Schematic representation of streamlines in the flow field near the drain.

From Fig. 5, the gradient along the streamline at the drain or ditch boundary is given by the expression:

$$d(\varphi)/ds = d(\varphi)/(dx + dy) = 1/(dx/d(\varphi) + dy/d(\varphi)) \tag{5}$$

To calculate the gradient one needs to determine the explicit relationships x as a function of φ and y, and y as a function of φ and x, respectively. These functions can be obtained from expressions (3) and (4). To facilitate the algebraic manipulations, we redefine the RHS of Eqs. (3) and (4) as $f_1(\varphi,\psi)$ and $f_2(\varphi,\psi)$, respectively. Then Eq. 3 yields the following explicit relationships for y and x:

$$Y = \frac{d}{\pi} \cdot \cos^{-1}\left[\frac{\cosh \frac{c}{d}\pi}{\cosh \frac{x}{d}\pi} - \frac{\left(\cosh \frac{c}{d}\pi - \cos \frac{b}{d}\pi\right)}{\cosh \frac{x}{d}\pi} \cdot f_1(\varphi,\psi)\right] \tag{6}$$

and

$$X = \frac{d}{\pi} \cdot \cosh^{-1}\left[\frac{\cosh \frac{c}{d}\pi}{\cos \frac{y}{d}\pi} - \frac{\left(\cosh \frac{c}{d}\pi - \cos \frac{b}{d}\pi\right)}{\cos \frac{y}{d}\pi} \cdot f_1(\varphi, \psi)\right] \tag{7}$$

Likewise, Eq. 4 yields the following explicit relationships for y and x:

$$Y = \frac{d}{\pi} \cdot \sin^{-1}\left[\frac{\cosh \frac{c}{d}\pi - \cos \frac{b}{d}\pi}{\sinh \frac{x}{d}\pi} \cdot f_2(\varphi, \psi)\right] \tag{8}$$

and

$$X = \frac{d}{\pi} \cdot \sinh^{-1}\left[\frac{\cosh \frac{c}{d}\pi - \cos \frac{b}{d}\pi}{\sin \frac{y}{d}\pi} \cdot f_2(\varphi, \psi)\right] \tag{9}$$

Mutual substitution of Eqs. 7 and 9 yields after several algebraic manipulations:

$$tg(\frac{y}{d}\pi) = \left[-\frac{(A-B-1)}{2A} \pm \sqrt{\frac{(A-B-1)^2}{4A^2} + \frac{B}{A}}\right]^{\frac{1}{2}} \tag{10}$$

Where, for a given ψ, A = A(φ)=(cosh (πc/d) - a.$f_1(\varphi,\psi)$)**2, B = B(φ)= (a.$f_2(\varphi,\psi)$)**2, and a = (cosh(πc/d) - cos(πb/d)). Eq. (10) represents an explicit relationship of y in terms of φ for a given ψ or streamline. The relationship dy/dφ can now readily be determined by straightforward differentiation. Likewise, mutual substitution of Eqs. 6 and 8 yields after several algebraic manipulations:

$$\sinh(\frac{x}{d}\pi) = \left[-\frac{(A+B-1)}{2} \pm \sqrt{\frac{(1-A-B)^2}{4} + B}\right]^{\frac{1}{2}} \tag{11}$$

where A and B are defined as before. Eq. (11) represents an explicit relationship of x in terms of φ for a given ψ or streamline. The relationship dx/dφ can now also be determined by straightforward differentiation.

Having those relationships (10) and (11) the gradient dφ/ds is now determined by virtue of Eq. 5 and the location of the gradient on the wetted perimeter is determined by virtue of Eqs. (1) and (2) bearing in mind the value of the streamline ψ in terms of a fraction of Q_1 and the potential function that represents the difference between the water levels in the field and the ditch adjusted for the hydraulic conductivity. Also, the angle of the exit gradient with the positive x-axis is determined from the ratio of dφ/dy and dφ/dx. The derivation of these quantities are algebraically quite involved but are, for this case, explicit and thus are readily amenable to straightforward programming and evaluations.

In evaluating $f_1(\varphi,\psi)$ and $f_2(\varphi,\psi)$ define $u = \exp(-\pi\varphi/Q_1)$ and substitute the quantity u into the RHS of Eqs. 1 and 2. One obtains, respectively:

$$f_1(\varphi,\psi) = \frac{u^4 - 4u^2 \cdot \sin^2 \frac{\pi\psi}{Q_1} - 2u^2 + 1}{u^4 - 4u^3 \cdot \cos \frac{\pi\psi}{Q_1} + 4u^2 \cdot \cos^2 \frac{\pi\psi}{Q_1} + 2u^2 - 4u \cdot \cos \frac{\pi\psi}{Q_1} + 1} \tag{12}$$

and

$$f_2(\varphi,\psi) = \frac{4u^3 \cdot \sin \frac{\pi\psi}{Q_1} - 4u \cdot \sin \frac{\pi\psi}{Q_1}}{u^4 - 4u^3 \cdot \cos \frac{\pi\psi}{Q_1} + 4u^2 \cdot \cos^2 \frac{\pi\psi}{Q_1} + 2u^2 - 4u \cdot \cos \frac{\pi\psi}{Q_1} + 1} \tag{13}$$

The expressions $df_1/d\varphi$ and $df_2/d\varphi$ are now readily determined from Eqs. 12 and 13 using the chain rule:

$$df_1/d\varphi = df_1/du \cdot du/d\varphi = df_1/du \cdot (-\pi\, u/Q_1) \tag{14}$$

and

$$df_2/d\varphi = df_2/du \cdot du/d\varphi = df_2/du \cdot (-\pi\, u/Q_1) \tag{15}$$

In this study, calculation will be made for the simple case of the drain model where c = 0.

SUMMARY

In this paper the calculation of the exit gradient of seepage into a ditch and drain for a rather simple flow field were derived. Because of the simplicity of this flow field exact expressions were obtained.

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