FLOW RESISTANCE IN OPEN CHANNELS WITH FIXED AND MOVABLE BED

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Abstract In spite of an increasingly large body of research by many investigators, accurate quantitative prediction of open channel flow resistance remains a challenge. In general, the relations between the elements influencing resistance (turbulence, boundary roughness, and channel shape features, such as discrete obstacles, bars, channel curvature, recirculation areas, secondary circulation, etc.) and mean flow variables are complex and poorly understood. This has resulted in numerous approaches to compute friction using many and diverse variables and equally diverse prescriptions for their use. In this paper, a new resistance law for surface (grain) resistance, the resistance due to the flow viscous effects on the channel boundary roughness elements, is presented for the cases of flow in the transition ($5 < Re* < 70$) and fully rough ($Re* \geq 70$) turbulent flow regimes, where $Re*$ is the Reynolds number based on shear velocity and sediment particle mean diameter. It is shown that the new law is sensitive to bed movement without requiring previous knowledge of sediment transport conditions. Comparisons between computation and measurements, as well as comparisons with other well-known existing roughness predictors, are presented to demonstrate its accuracy and range of application. It is shown that the method accurately predicts total friction losses in channels and natural rivers with plane beds, regardless of sediment transport conditions. This work is useful to hydraulic engineers involved with the derivation of depth-discharge relations in open channel flow and with the estimation of sediment transport rates for the case of bedload transport.

INTRODUCTION

The problem of determining the flow velocity and depth in a channel, for a known discharge, remains an often revisited topic in fluvial engineering. One of the principal causes is the difficulty in determining accurately flow-resistance coefficients. Among the diverse and complex phenomena influencing resistance are turbulence, boundary roughness, and channel shape features, such as discrete obstacles, bars, channel curvature, recirculation areas, secondary circulation, etc. The presence of bedload is also known to have direct impact on flow resistance. For example, adding sediment to a clear water flow continually increases the resistance until the carrying capacity is fulfilled. This is easy to understand, as energy from the flow is then used to move sediment, which is extracted from the flow and results in a decrease in velocity and an increase in apparent roughness height that is comparable to the thickness of the moving sediment layer.

One of the commonly used formulas to relate mean flow to friction in open-channel flow is the Darcy-Weisbach equation:

$$\sqrt{\frac{8}{f}} = \frac{U}{u*} = \frac{U}{\sqrt{gRS}} \tag{1}$$

where f = Darcy-Weisbach friction factor; U = mean flow velocity; $u*$ = shear velocity; g = acceleration due to gravity; R = hydraulic radius; and S = energy slope. A widely adopted concept for expressing f in rough turbulent flow is Nikuradse's equivalent grain roughness, k_s, and the Prandtl-von Karman velocity distribution law (Keulegan, 1938):

$$\frac{U}{u*} = C_0 \log \frac{12.2R}{k_s} \tag{2}$$

where the constant $C_0 = 2.3/\kappa$ ($\kappa = 0.40$ is von Kármán's coefficient in open channel flow). Combining eqs. (1) and (2) yields the well-known expression

$$\sqrt{\frac{8}{f}} = 5.75 \log \frac{12.2R}{k_s} \tag{3}$$

In plane-bed roughness, k_s is assumed to be proportional to a characteristic size of the wall roughness elements which, in fluvial hydraulics, comprise the sediment particles present in the channel's bed:

$$k_s = \beta_i d_i \tag{4}$$

where d_i is the particle diameter of the ith percentile belonging to size fraction i in the bed material (e.g., d_{50} is the diameter of the 50 percentile of the particle size distribution, i.e., its median diameter) and β_i is the coefficient associated with it. One of the problems raised with this formulation lies in the choice of characteristic particle diameter in the case of graded sediments, which is not a trivial matter and has resulted in a multitude of published approaches (see Table 1). Furthermore, it is reasonable to expect that two different gradations with the same d_i but different gradation coefficient would have different β_i values.

Table 1 Some of the values of $k_s = \beta_i d_i$ used by different investigators. The list is not complete, but provides a perspective of the range of β_i for different d_{50} found in the literature.

d_i	β_i	Reference	d_i	β_i	Reference
d_{35}	1.23	Ackers and White (1973)	d_{75}	1.0	Kleinhans and van Rijn (2002)
d_{50}	1.0	Keulegan (1938)		3.2	Lane and Carlson (1953)
	4.5	Thompson and Campbell (1979)	d_{84}	1.5	Ikeda (1983)
	5.0	Griffiths (1981)		3.5	Hey (1979)
d_{65}	1.0	Einstein and Barbarossa (1952)		5.1	Mahmood (1971)
	2.0	Engelund and Hansen (1967)	d_{90}	0.5	Kleinhans and van Rijn (2002)
				3.0	Van Rijn (1982)

Another limitation of eq. (4) is its insensitivity to sediment transport and to the presence of bedload, which has the effect of increasing bed resistance to the flow. This effect has been shown by Recking et al. (2008), among others, and is presented here in Fig. 1. The data shows a general tendency for an increase in flow resistance in the presence of sediment movement, with corresponding increase in apparent grain roughness, but there is considerable deviation from the resistance law represented by eq. (3) when eq. (4) is used to characterize bed roughness.

The objective of the study presented herein is to provide an analysis of the basic resistance law in eq. (2) for plane-bed flows with and without sediment transport. There is the desire to retain Nikuradse's concept of equivalent grain roughness, but it is shown that the current formulation represented in eq. (4) is insufficient. An alternative method is proposed, where flow parameters are used as a feedback mechanism to calculate k_s. In other words, the constant parameter β_i of eq. (4) is replaced by varying functions that include not only sediment diameters, but also flow-dependent quantities that are better able to reflect the changes in hydraulics that are related to sediment transport in general, and in particular to bedload. Finally, the new method is compared to other existing methods and is applied to laboratory channels and natural streams to show its validity and range of applicability.

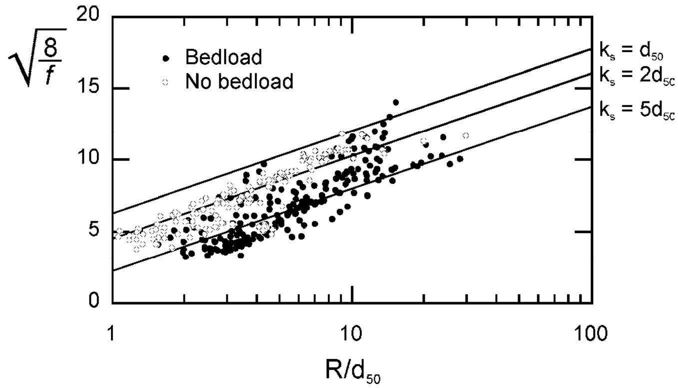

Figure 1 Measured flow resistance in flat-bed channels with and without bedload. The lines represent eq. (3) with different expressions for k_s. The circles are experimental data from Cao (1985), Smart (1984) and Recking et al. (2008).

DATA ANALYSIS

Preliminary analysis of some of the data available in the literature clearly shows the limitation of the formulation represented in eq. (4). This point is made clear in Fig. 2, where Keulegan's original formulation is used to estimate the friction factor for a number of different flow conditions. In particular, note the significant underprediction of the friction factor for larger values of *f*. Some of this disagreement may be attributed to the presence of bedload and the inability of the present method to capture that effect. Keulegan's relation ($k_s = d_{50}$) is particularly interesting for this work because d_{50} is the most likely known sediment particle size quantity, therefore it is desirable to use it over other characteristic particle diameters.

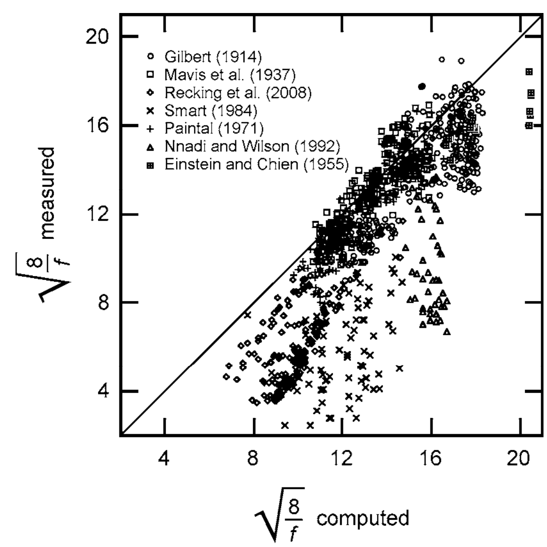

Figure 2 Comparison between measured and predicted Darcy-Weisbach friction factor using $k_s = d_{50}$ and plane-bed data (grain resistance only). The solid line is the line of perfect agreement.

It has been argued that, in mobile beds, bed material movement and entrainment in the flow contribute to a higher apparent roughness height that is proportional (or, at least, related) to the thickness of the bed layer in motion. At least for the case of sheet flow, which occurs for high values of the Shields parameter θ, the thickness of the sheet flow layer is much larger than the sediment particle sizes and has been shown to be related to both d_{50} and θ (Wilson, 1987). The dimensionless Shields parameter is defined as $\theta = \frac{\tau_c}{(\rho_s - \rho) g d_{50}}$ where τ_c = bed-shear stress, and ρ, ρ_s = density of water and sediment, respectively.

Analysis of the high-shear data used in this study suggested a relation of the type $k_s \propto d_{50}^a \exp(-\theta)$, where *a* is a coefficient to be determined—see Fig. 3. The data presented in Fig. 3 suggests *a* = 2, but this is undesirable because it would result in dimensional fit coefficients, therefore the product $d_{50} d_{50}^*$ is used instead, where d_{50}^* is the dimensionless mean grain diameter defined as $d_{50}^* = d_{50} \left(\frac{g(s-1)}{\nu^2} \right)^{1/3}$ where ν = kinematic viscosity of water and *s* = specific density of sediments (= ρ_s/ρ). With these considerations, a nonlinear least-squares fit to the data corresponding to θ ≥ 1 was done to the expression

$$k_s = d_{50} d_{50}^* \left(a + b\theta + c \exp(-\theta) \right) \tag{5}$$

yielding *a* = -0.716, *b* = 0.473, and *c* = 0.920. This fit has a Pearson's correlation coefficient *R* = 0.982.

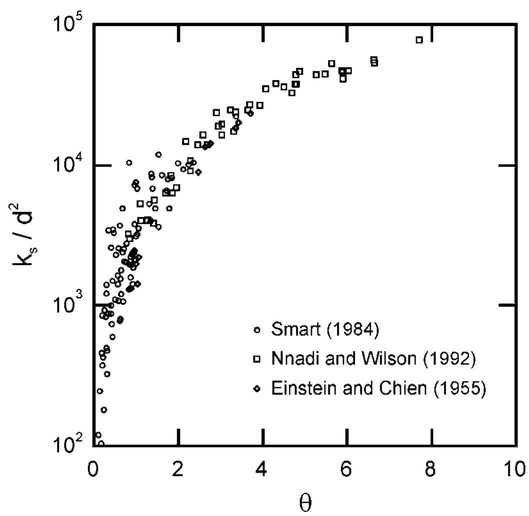

Figure 3 Apparent roughness height for high shear data.

Analysis of the 1,097 sets of data used in Fig. 2 indicates that k_s behaves differently for different ranges of θ. The data were grouped in two regions and a multivariate nonlinear least-squares analysis and fit was carried out separately for each region. The resulting equations for k_s are, for SI units,

$$k_s = 1.08 d_{50}^{0.968} 5390^{\theta} \quad \text{for} \quad \theta < 0.2 \tag{6}$$

$$k_s = \exp\left(-10.5R + 70.6 d_{50} + 10.5S - 4.49 \right) \quad \text{for} \quad 0.2 \le \theta < 1 \tag{7}$$

These two fits yield a correlation coefficient R^2 = 0.83 for eq. (6) and R^2 = 0.85 for eq. (7). It is also convenient to note that the above two equations are expressions that were found to fit the data well, but have no intrinsic physical significance.

The previous analysis was carried out for data in the fully rough regime, i.e., data for which the Reynolds number $Re^* \geq 70$, where $Re^* = u^* d_{50}/\nu$. Flow in the transition regime ($5 < Re^* < 70$) is traditionally calculated as a blend of the smooth and rough treatments, i.e. eq. (3) is replaced by

$$\sqrt{\frac{8}{f}} = 5.75 \log\left(\frac{12.2h}{k_s + 3.3\dfrac{\nu}{u^*}}\right) \tag{8}$$

However, in spite of trying the different prescriptions of k_s presented in Table 1, it was observed that eq. (8) does not represent well the data used in this study. Several parameters were used to represent k_s in the transition regime, but the scatter of the data seems to indicate that Keulegan's concept of apparent roughness is not readily applicable here. In order to extend the computation to the transitional regime, the friction factor f was directly fit to the data based on the product θR which was observed to provide an acceptable independent variable. The resulting equation is

$$\sqrt{\frac{8}{f}} = a + b\theta R + \frac{c}{\sqrt{\theta R}} \tag{9}$$

with a = 16.2, b = 3.03, and c = -0.117. SI units are used in eq. (9). The comparison between this expression and the experimental data is shown in Fig. 4. Once again, eq. (9) is simply a data fit and is not based on any physical considerations. Eqs. (5), (6), (7), and (9) constitute a set for the calculation of the Darcy-Weisbach friction factor for turbulent plane-bed open channel flow where grain resistance is the dominant effect.

VERIFICATION AND APPLICATION

Application of the method described in the preceding section is straightforward and can be easily implemented in a computer program. The steps are the following:

1. Compute u^* ($= \sqrt{gRS}$) and Re^*.
2. If $Re^* < 70$ use eq. (9) and go to step #8.
3. Compute θ.
4. If $\theta < 0.2$ use eq, (6) and go to step #7.
5. If $0.2 \leq \theta < 1.0$ use eq. (7) and go to step #7.
6. If $\theta \geq 1.0$ use eq. (5).
7. Compute $\sqrt{8/f}$ using eq. (3).
8. Compute U (or the discharge Q) using eq. (1).

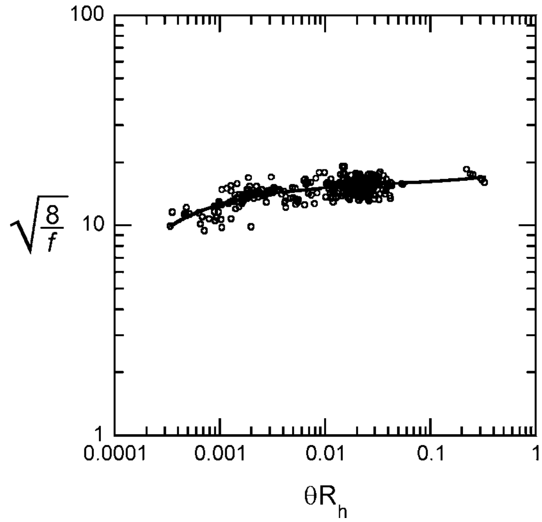

Figure 4 Friction factor for data in the transition regime. The solid line is eq. (9) and R_h is the hydraulic radius.

Application of the algorithm to the original data of Fig. 2 is presented in Fig. 5. The data points are labeled to show the result of the application of the different equations—eqs. (5), (6), (7), and (9)—in their respective ranges of applicability. Note the significant improvement in the prediction of the Darcy-Weisbach friction factor, particularly in the region of high *f*, where sediment transport is most likely to occur with the highest intensity.

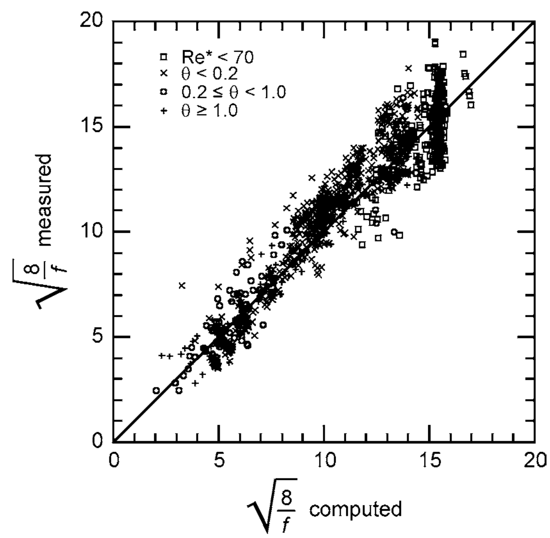

Figure 5 Application of the new computation procedure to the data of Fig. 2.

Camacho and Yen (1992) have developed an approach to compute fluvial resistance by using the Froude number *Fr* as a classification parameter. In their approach, the data are divided in four regions ($Fr < 0.4$, $0.4 \leq Fr < 0.7$, $0.7 \leq Fr < 1$, and $Fr \geq 1.0$) and fitted by different expressions using d_{50}, θ, *R*, and *Fr* as independent variables. To provide a means of comparison with the present algorithm, the Camacho and Yen (1992) approach was applied to the same data of Fig. 2 and the result is presented in Fig. 6. Significant discrepancy between computation and measurement is apparent. The limitation of the Camacho and Yen (1992) algorithm is further underlined by the fact that only 797 data points are shown due to the fact that Camacho and Yen's algorithm requires an iterative computation procedure and failed to converge for 300 sets of data.

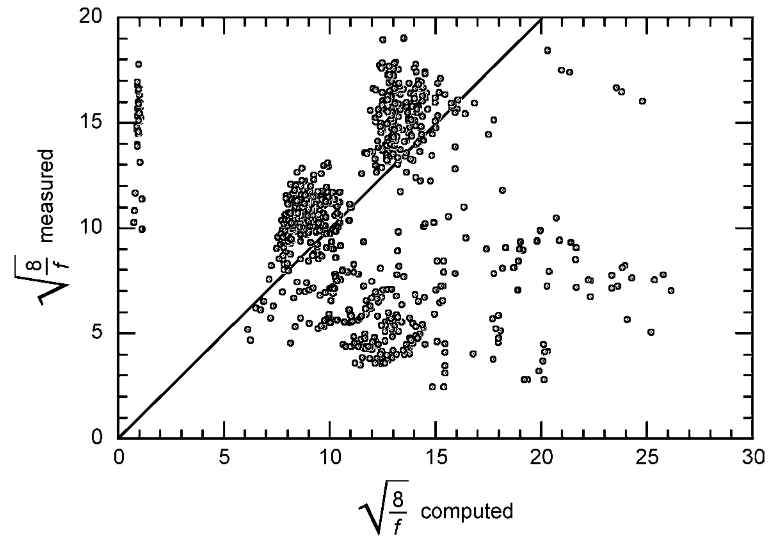

Figure 6 Camacho and Yen (1992) method applied to the data of Fig. 2. The solid line is the line of perfect agreement.

The verification of the method presented in this study is now carried out using both field and laboratory data. These data sets, 94 from riprap data, 355 from field data, and 218 from laboratory data, are: the riprap data from Maynord (1991); the laboratory data of Cao (1985) and Graf and Suszka (1987); the American Canal data of Simons (1957); the Mountain Creek data of Einstein (1944); the Rio Grande (near Bernalillo, New Mexico) data of Toffaleti (1968); and data from Johnson Creek (Yellow Pine, Idaho), Lochsa River (Lowell, Idaho), South Platte River (Buffalo, Colorado), Wisconsin River (Muscoda, Wisconsin), and Yampa River (Deerlodge Park, Colorado) collected from USGS reports. The proposed calculation algorithm was used to compute $\sqrt{8/f}$, followed by eq. (1) to compute the velocity U. The result is shown in Fig. 7, where the solid lines represent perfect agreement and the dashed lines are the ±20% bounds.

Fig. 7 shows that the agreement between computed and measured mean flow velocities is close for both laboratory and field data, as most of the data falls within the 80% confidence interval (i.e., the predicted values have less than 20% error). However, the fit is much better in the case of laboratory and riprap data, with nearly 95% of this data in the 80% confidence interval. This is not surprising, because the dominating effect in these flows is grain resistance—especially in the case of riprap—which are the prevailing assumptions in the derivation of the method presented in this work. The field data was measured under less controlled conditions, therefore effects other than grain roughness may play important role in the overall total (measured) roughness. This can be seen by the larger data spread, an extreme case of which is represented by the South Platte data.

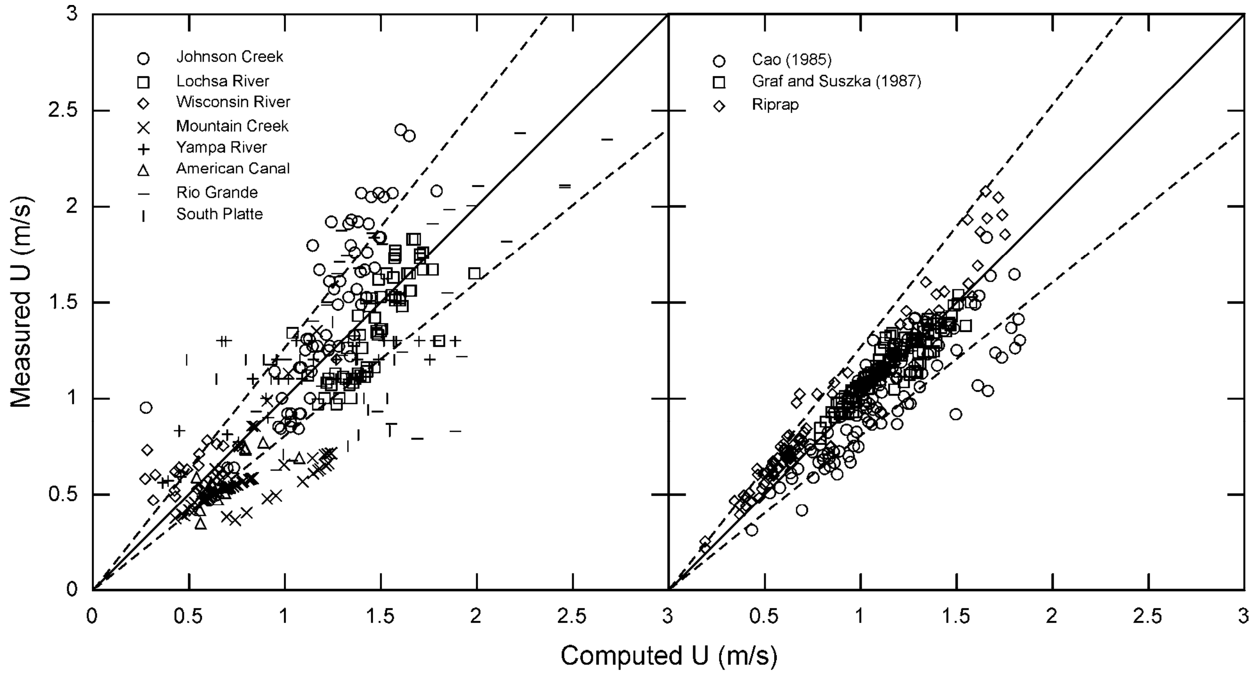

Figure 7 Agreement between computed and measured flow velocity for field (left) and laboratory (right) validation data. The solid line is the line of perfect agreement and the dashed lines show the ±20% error bounds.

The poorest predictions concern the Mountain Creek and the Wisconsin River data, which have a large percentage of points falling outside of the 80% confidence interval. Unfortunately, the literature did not provide indication of presence or absence of bed forms, which may be responsible for this deviation. The method presented in this work is applicable to plane-bed grain roughness only and more research is needed to determine the reason for the shown discrepancy. However, it is convenient to point out that alluvial resistance may be linearly separated as

$$f = f' + f'' \tag{10}$$

where f = total flow resistance. In this case, f' is calculated using the algorithm proposed here (plain-bed resistance) and f'' is calculated taking into consideration the additional factors contributing for overall resistance (bed-forms, for example). Nonetheless, the overall agreement between computation and measurements was close for the cases where bed forms were known to be absent. In particular, the validation data contains data points with and without bedload transport, and the proposed method is able to predict both conditions without the need for additional information.

Additional insight can be drawn by comparing existing and well established methods to the same data of Fig. 7, providing a direct comparison between those methods and the one proposed in this article. Here, the well-known formulation of Yalin (1992) is used, as formulated by Yang and Tan (2008). In this formulation, $k_s = 2d_{50}$ and

$$\sqrt{\frac{8}{f}} = 2.5\ln\left(\frac{h}{k_s}\right) + B \tag{11}$$

where h = water depth and B is expressed as

$$B = 6 + \left(2.5\ln k_s^+ - 3\right)\exp\left(-0.11(\ln k_s^+)^{2.5}\right) \tag{12}$$

with $k_s^+ = u^* k_s/\nu$. The goodness-of-fit of the formulation of eq. (11) for laboratory and riprap data is shown at the right-hand side of Fig. 8. Although the overall agreement is still acceptable for laboratory data—it is very good for the data of Graf and Suszka (1987)—a significantly larger number of data points now falls outside the 80% confidence interval. In particular, the riprap data has now a larger spread around the line of perfect agreement. As shown in Fig. 8, there is an overall increase in the scatter of the data points and in the proportion falling outside the ±20% limits, resulting in an overall degradation of the fit relative to the proposed algorithm.

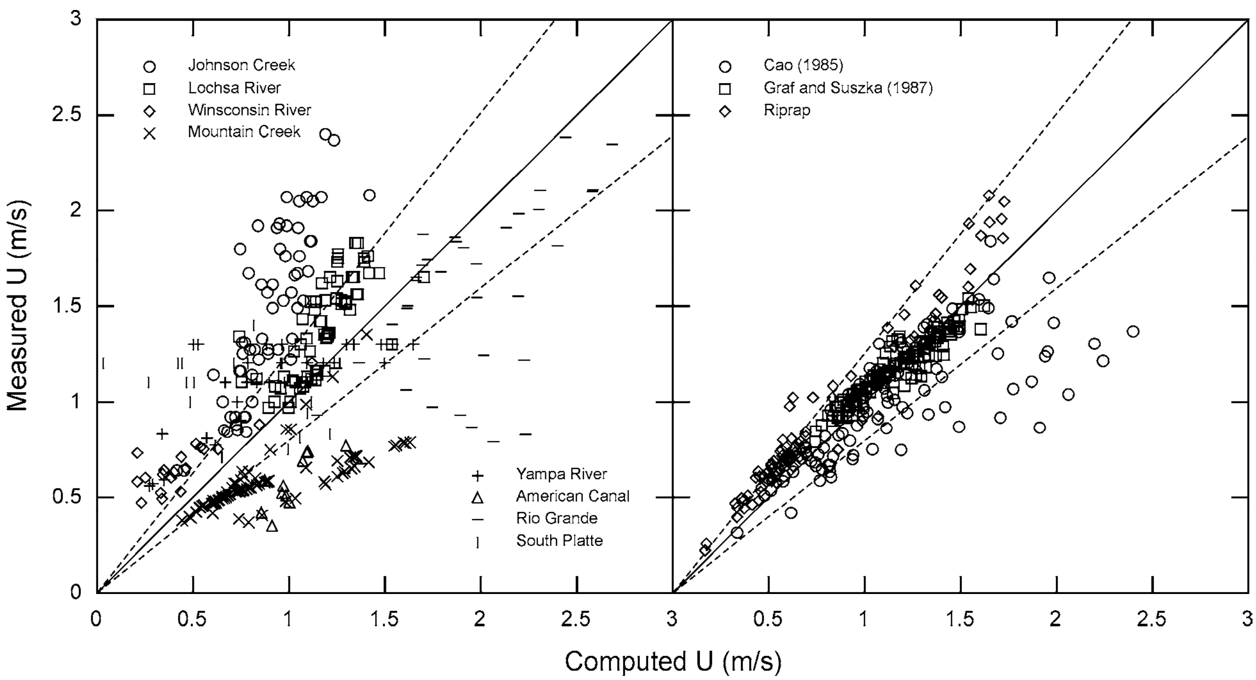

Figure 8 Agreement between measured and predicted flow velocity for field (left) and laboratory (right) data using a modified Yalin (1992) formulation. Solid line: perfect agreement; dashed lines: ±20% error bounds. The data are the same used in Fig. 7.

For the field data, however, the agreement between measured and computed velocity is very poor, with most of the data points falling outside of the 80% confidence interval. This may be attributed to the larger variability and complexity associated with field conditions. Nonetheless, it is clear that the proposed method based on variable k_s is superior in modeling the data used, providing significantly better predictions of bed friction and velocity.

CONCLUSION

The determination of alluvial roughness is a challenging problem in fluvial hydraulics. A widely used approach to this problem is provided by Keulegan's concept of apparent grain roughness. Unfortunately, this concept seems to suffer from lack of universality—indicated by the many alternative formulations used in the literature (Table 1)—and from the inability to account for the difference between mobile and immobile beds, as indicated by the static form of eq. (4). Nonetheless, it remains an attractive concept due to its readily accessible physical interpretation. In a preceding section, the concept of k_s was expanded to include high shear transport—eq. (5)—and to approximate data by sectioning it based on the Shields parameter θ. Together with the median sediment particle diameter d_{50}, the proposed method added the use of hydraulic variables that are known to be associated to sediment transport: θ and S. Dependence on θ was shown to be particularly significant for high shear data, where the apparent roughness is related to the thickness of the sheet flow layer, which in turn is proportional to θ. This allowed the sensitization of k_s to the effects of bedload.

The concept of grain roughness is valid only for turbulent flows in the fully rough regime ($Re^* \geq 70$). In order to extend the proposed calculation procedure to transition flows ($5 < Re^* < 70$), Keulegan's concept had to be abandoned and replaced by a direct fit of the Darcy-Weisbach roughness coefficient to laboratory data. The resulting set of equations was assembled in a calculation algorithm that was shown to be able to significantly improve the ability to predict plane-bed flow with and without sediment transport.

A good fit between computed and measured flow velocities using both field and laboratory data provides validation of the method presented in this paper. Comparison with other existing roughness predictors also showed significant improvement in the accuracy of the predictions obtained by this method. Nonetheless, it is important to recognize its limitations: derivation of the equations was carried out using only laboratory data with plane beds made of sand and gravel, with both uniform and graded sediments. It included fixed and moving beds, but properties of the sediment particle distributions, such as gradation coefficients, were ignored. This does not mean that they do not play an important role (probably they do). Instead, it was decided to use only the mean particle diameter d_{50} because this is the most probable and easy quantity to know, resulting in an algorithm that is simple to apply using common and readily available hydraulic parameters.

Finally, in spite of the possibility of using a compartmented approach to total flow resistance as implied by eq. (10), it would be desirable to extend the proposed methodology to bed-form resistance. More research is also desirable to replace eqs. (6), (7), and (9) with physically meaningful relations. Nonetheless, the algorithm presented can be used to generate depth-discharge relations for many natural rivers and is useful to hydraulic engineers involved with open channel flow and with the estimation of sediment transport rates.

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