# MODELING LONG-TERM SEDIMENT BUDGETS IN SUPPLY-LIMITED RIVERS

**Scott A. Wright, research hydrologist, USGS, Sacramento, CA, sawright@usgs.gov; David J. Topping, research hydrologist, USGS, Flagstaff, AZ, dtopping@usgs.gov; David M. Rubin, research geologist, USGS, Santa Cruz, CA, drubin@usgs.gov; Theodore S. Melis, physical scientist, USGS, Flagstaff, AZ, tmelis@usgs.gov**

## INTRODUCTION

Engineers and geomorphologists often need to simulate changes in fluvial sediment budgets resulting from changes in driving forces, such as climate, dam operations, land-use changes, etc., in support of resource management. Humans have had a dramatic impact on the world's river systems (Nilsson et al., 2005), including sediment transport and budgets (Syvitski et al., 2005). Because sediment provides the physical framework for aquatic ecosystems, management of aquatic resources requires the ability to simulate changes in sediment budgets resulting from natural and anthropogenic influences.

In response to this need, substantial research and development has been conducted in the area of fluvial sediment transport modeling. The wide range of space and time scales of interest has lead to a range of modeling approaches, from simple empirical "rating curve" approaches to one-dimensional morphologic models (e.g. HEC-6, Rahuel et al., 1989, van Niekirk et al., 1992, among many others) to complex multi-dimensional models (e.g. Deflt3D). Suspended-sediment rating curves (i.e. empirical relations between suspended-sediment concentration and water discharge) are an attractive approach for evaluating long-term sediment budgets because they are very simple, easy to implement computationally, and the empirical parameters can be estimated from quantities that are frequently measured in the field. However, an implicit assumption in this approach is that sediment transport is in quasi-equilibrium with sediment supply. Rubin and Topping (2001) presented an approach for evaluating this assumption for sand-bedded rivers and suspended-sand transport, and showed that sediment supply is often as important as water discharge in regulating sand transport.

Because of the physical limitations of the rating curve approach and the computational limits of more complex models when applied to long reaches over long time scales, we have developed and tested an alternative approach that bridges this gap. The approach uses empirically-based rating curves, but they are formulated on a particle-size specific basis that always for simulation of changes in bed particle size distributions, albeit in a simplified manner. Thus, the rating curves can respond to changes in sediment supply with a formulation that is quite simple, computationally efficient, and easy to implement. To further facilitate long-term simulations of multiple scenarios, the channel is idealized in shape and spatially discretized over long reaches (10s of km). In this paper, we present this modeling approach and its application to the Colorado River below Glen Canyon Dam. We do not argue that the empirical parameters developed for the Colorado River have general applicability; rather, they are site-specific. However, the modeling approach should have general applicability where evaluation of long-term sediment budgets is of interest, such as below dams where the flows and sediment supply are often dramatically altered.

## STUDY SITE

The modeling approach described in the next section was developed as part of our ongoing work on the Colorado River below Glen Canyon Dam (fig. 1). The construction of Glen Canyon Dam in the early 1960s substantially reduced the supply of sand to the downstream reach (Topping et al., 2000) as well as the capacity of the river to transport sand by reducing large flood peaks (Topping et al., 2003). The post-dam flow regime is illustrated in fig. 2 which shows the study period over which the model was applied. For a complete review of pre- and post-dam flow regimes refer to Topping et al. (2003). Note that we use the English unit for water discharge (cubic feet per second or cfs) herein because of its common use and acceptance within the Colorado River scientific, management, and recreational community.

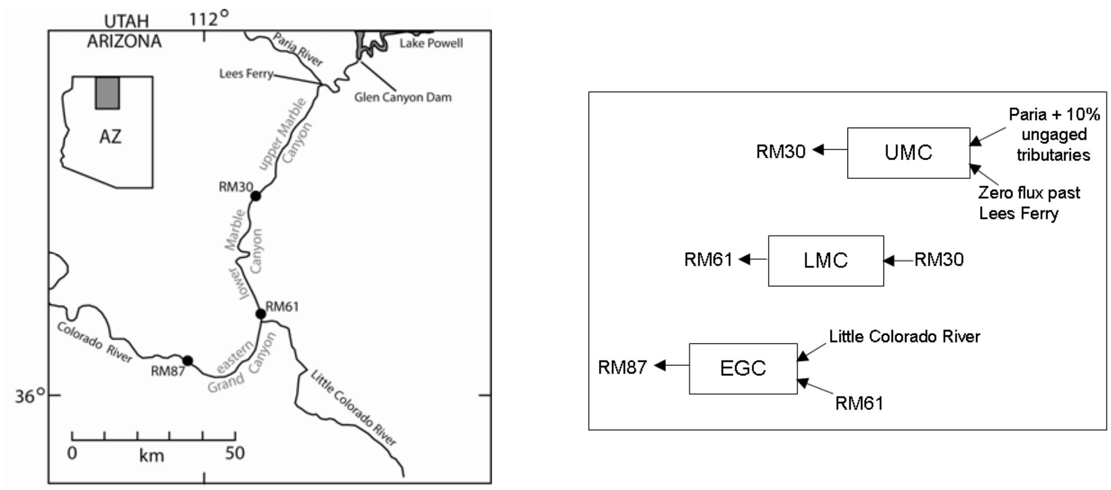


Figure 1 Left – Location map of the study reach of Colorado River below Glen Canyon Dam. Right – Modeling schematic showing the reaches and tributary input locations.

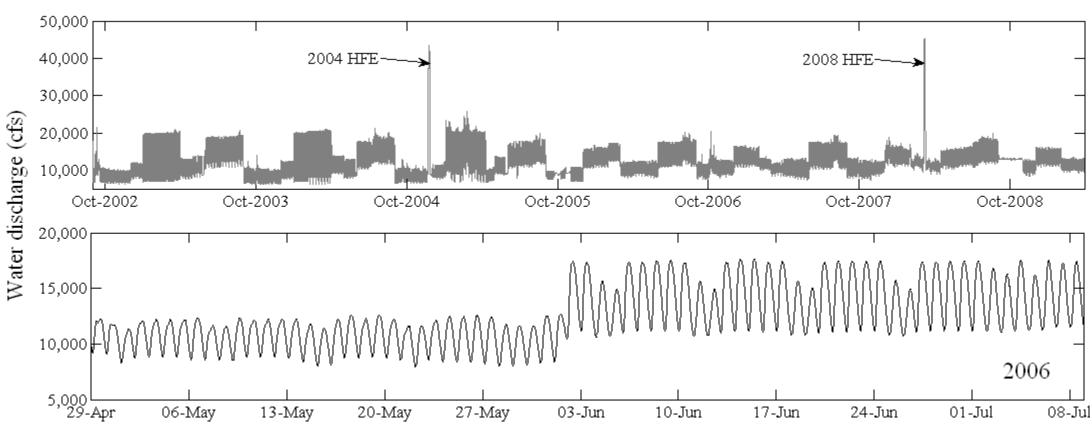


Figure 2 Top – Discharge time series for the RM87 gage for the entire modeling period. Bottom – Illustration of daily hydropower fluctuations in the spring and summer of 2006.

The post-dam sand budget is important to the fate of sandbars along the river. These sandbars are considered a valued resource within the Glen Canyon Dam Adaptive Management Program, a federal advisory committee established to provide guidance on operation of Glen Canyon Dam, due to their recreational, biological, and cultural importance (Lovich and Melis, 2005). Several previous studies of the post-dam sand budget have come to conflicting results (Wright et al., 2005), but a review of available data indicates that sandbars have been substantially eroded since construction of the dam (Schmidt et al., 2004). The strategy to abate this erosion has been the release of short-duration “controlled floods” or “high-flow experiments” (HFEs) designed to redistribute sand from subaqueous channel pools to subaerial sandbars (e.g. Wright et al., 2008). Three of these experiments have been conducted (in 1996, 2004, and 2008) and the results indicate that substantial sandbar deposition is possible when the channel is sufficiently enriched with sand sourced from tributaries downstream from the dam. Thus, the ability to simulate sand budgets for various operations is important toward designing and implementing future HFEs.

The approach presented herein was designed specifically to account for the supply limitation that is known to exist in this reach of river. For example, Topping et al. (1999, 2000) showed that sand transport rates are strongly dependent on tributary sand supply as well as water discharge. This dependence is illustrated in fig. 3 which shows changes in the relation between suspended-sand concentration and water discharge resulting from a major flood on the Paria River (the first major tributary downstream from the dam, fig. 1). It is seen that concentrations (for a given discharge) are much greater during tributary flooding and remain significantly higher than pre-flood levels after the flooding recedes. We note that several previous modeling approaches have been applied to this study site, but they either cannot account for changes in sediment supply (e.g. Randle and Pemberton, 1987) or are substantially more complex and thus not suitable to our modeling goals, as defined in the next section (e.g. Wiele et al., 1996, 1999, 2007).

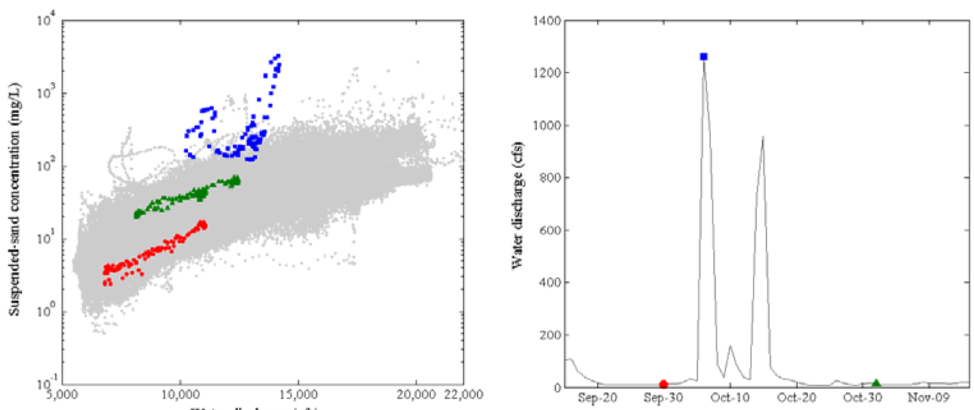


Figure 3 Left – SSC versus discharge for the RM30 gage (all data - gray dots) highlighting three days before (red circles), during (blue squares), and after (green triangles) Paria River flooding. Right – Paria discharge record showing the dates of the highlighted data.

## MODELING APPROACH

We outlined several overarching goals for the modeling approach as applied to our study site: 1) the model should be simple enough to allow for long-term (annual to decadal scale) simulations and the ability to model a large number of scenarios to account for variability in hydrology, sediment supply, and operations; 2) the model should reproduce the basic processes of sand accumulation and fining of the bed during and immediately after tributary flooding, followed by erosion and bed coarsening during tributary quiescence; 3) the number of empirical model parameters should be as few as possible and readily determined from available data. The approach described below was designed to achieve these goals.

Rubin and Topping (2001) applied the suspended-sediment transport formulation of McLean (1992) to a wide range of hydraulic conditions and particle size distributions and found that concentration can be approximated as a power-law function of shear velocity ($u_*$) and bed particle size ($D_b$), with the $u_*$ exponent ranging from 3.5 to 5 and the $D_b$ exponent ranging from -1.5 to -3.0, depending on bed conditions (i.e. dunes present, bed sorting coefficient). If $u_*$ is approximated by a power-law function of discharge then the relation of Rubin and Topping (2001) can be rewritten in the form of a size-specific sediment rating curve:

$$C_i = F_{bi} A Q^L D_i^K \tag{1}$$

where $C_i$ is concentration of size *i*, $F_{bi}$ is the fraction of size *i* in the bed material, *A* is an empirical site-specific constant, *Q* is water discharge, and $D_i$ is diameter of size *i*. Note that eq. 1 reduces to the standard sediment rating curve form when summed over all sizes because $\sum F_{bi} = 1$ and $\sum D_i^K = const$. To apply eq. 1, *A*, *L*, and *K* must be estimated empirically on a site-specific basis; the advantage is that these parameters can be estimated from measurements of *C* and *Q*, two quantities that are routinely measured on many rivers ($C = \sum C_i$).

To apply eq. 1 in a modeling framework, a method is required for updating the bed material composition $F_{bi}$. For this we apply the active layer form of the Exner equation for bed sediment mass conservation (e.g. Parker et al., 2000), in a slightly simplified form. For our study site, which is a bedrock controlled canyon river, we assume that the mobile bed sediment is a thin layer of sand overlying bedrock; this assumption is supported by data presented in the following section. By assuming that the bedrock substrate is non-erodible and that the sand layer thickness ($H_s$) is equivalent to the active layer thickness, the Exner equation reduces to:

$$\left(1-\lambda_p\right) B \frac{\partial H_s}{\partial t} = -\frac{\partial Q_s}{\partial x} \tag{2}$$

$$\left(1-\lambda_p\right) B \frac{\partial}{\partial t}\left(H_s F_{bi}\right) = -\frac{\partial Q_{si}}{\partial x} \tag{3}$$

where $Q_s = CQ$, $Q_{si} = C_i Q$, *B* is channel width, and $\lambda_p$ is bed porosity. This formulation provides significant simplification over the standard Exner equation because it circumvents the need to keep track of substrate layering and associated size distributions.

The set of eqs. 1-3 constitutes a model for $C_i$, $H_s$, and $F_{bi}$ (assuming a separate hydraulic model supplies *Q*). The boundary conditions are $Q_{si}$ at the upstream boundary and major tributaries; required initial conditions are $H_s$ and $F_{bi}$ for each reach. The final approximation of our modeling approach is that we spatially discretized over relatively long reaches (10s of km). This assumption sacrifices the ability to model local channel complexity but allows for simulating the longer-term effects of bed fining and coarsening at substantially reduced computational costs.

We applied the formulation described above to three reaches bracketed by the existing gaging stations. The gage locations and modeling reaches are shown fig. 1 (geographically in the left panel, schematically in the right panel) and are defined as follows: Reach 1 (upper Marble Canyon, UMC) – Lees Ferry/Paria River confluence to RM30; Reach 2 (lower Marble Canyon, LMC) – RM30 to RM61/Little Colorado River confluence; and Reach 3 (eastern Grand Canyon, EGC) – RM61/Little Colorado River confluence to RM87. For the model applications, upwind finite differences were used for eqs. 2-3, with the following specifications: 15-min time step, 10 particle sizes spaced logarithmically between 0.0625 – 2 mm, $B = 80$ m, and $\lambda_p = 0.4$.

## ESTIMATION OF MODEL PARAMETERS

The data necessary to implement the modeling approach include time series of suspended-sand concentration and water discharge at RM30, RM61, RM87, time series of tributary sand inputs by size fraction (Paria and Little Colorado), initial sand thickness on the bed by reach, and initial bed particle size distributions by reach. Available data (described below) dictate that the modeling study period extends from Sep-2002 through Mar-2009.

The primary boundary condition requirements are size-specific sand fluxes from the major tributaries, the Paria and Little Colorado Rivers (mainstem sand flux past Lees Ferry is small in comparison due to erosion of background storage, Grams et al., 2007). For the Paria River, water discharge records (USGS gage 09382000) were used to estimate suspended-sand transport using the model developed by Topping (1997). For the Little Colorado River, data from two USGS gages (09402000 and 09402300) were used to estimate sand transport rates using a suspended-sand rating curve. The tributary sand size distributions were estimated by averaging available samples yielding roughly log-normal distributions with $D_{50}$ = 0.1 mm and $\sigma_g$ = 1.8. An additional 10% of the Paria inputs were added to UMC to account for ungaged tributaries (based on data from several ungaged tributaries). Finally, water discharge time series at each site were available from continuous stage measurements and stage-discharge relations.

Initial condition requirements include sand thickness on the bed and bed size distributions, for each of the three reaches. To estimate these quantities, we used data from remote sensing, ground surveys, bathymetric surveys (Kaplinski et al., 2007, 2009, Hazel et al., 2008), and bed particle size measurements (Rubin et al., 2007) for several 3-5 km reaches from 2000 – 2005. The measurement reaches constitute a relatively small percentage of the modeling reaches and thus are not guaranteed to be representative; however, they are the best available data for specifying the initial conditions. The reach surveys from May 2002 were used to estimate initial sand thicknesses of 0.4, 0.5, and 0.5 m and initial mean particle sizes of 0.4, 0.3, and 0.3 mm for UMC, LMC, and EGC, respectively (M. Breedlove, Grand Canyon Monitoring and Research Center, written communication) The bed size distributions were assumed log-normal with $\sigma_g = 2.0$ (estimated from available grab samples from the gage locations).

Using these data, the three empirical model parameters (*A*, *L*, and *K*) were estimated as follows. The *C*-*Q* exponents (*L*) were determined using data from an extensive suspended-sediment monitoring program that applies a combination of standard USGS techniques and "surrogate" technologies (acoustics, laser diffraction) to estimate concentration at 15-min intervals (Topping et al., 2007a, data are available at http://www.gcmrc.gov/products/other_data/). The exponents for each gage were estimated by curve fitting to data from the rising and falling limbs of the 2004 and 2008 HFEs (fig. 2) since these events encompass the full range of discharge over the study period and are of short duration such that the effects of changes in supply should be minimal. The data indicate a break in the curve for each site at about 25,000 cfs (Randle and Pemberton, 1987, also noted this break), and we have incorporated this aspect such that $L$ = {3.7, 4.0, 3.7} for RM30, RM61, RM87 respectively for $Q < 25{,}000$ cfs and $L$ = {1.7, 1.7, 1.3} for $Q > 25{,}000$ cfs. The particle size exponents (*K*) are difficult to estimate empirically because reach-based bed and suspended particle size data would be needed. However, Rubin and Topping (2001) reported a range of computed exponents of -1.5 to -3.0, thus providing a range of reasonable values. We conducted exploratory simulations and determined that a value of $K$ = -3.0 (for all gages) provided superior results with respect to the degree of fining and coarsening that has been observed (particularly during the high flow releases, next section). Finally, because the ultimate goal of our modeling is to simulate the long-term sand budget for the individual reaches, we chose to specify *A* at each gage location to match the measured total sand flux from the reach over the modeling period. With *L* and *K* specified as described above, these computations proceeded in a downstream direction whereby *A* was varied until the total sand flux from the reach matched the measured sand flux to within 1%. This yielded $A = \{3.6\times10^{-26},\ 6.6\times10^{-27},\ 6.0\times10^{-26}\}$ for RM30, RM61, RM87 respectively. When used in eq. 1, these coefficients result in $C_i$ as a volumetric concentration for $Q$ in m$^3$/s and $D_i$ in m.

## ANALYSIS AND DISCUSSION OF RESULTS

Figure 4 shows the results of calibrating the coefficient *A* so that the modeled sand flux from each reach matches the measured sand flux (±10% uncertainty) over the entire modeling period.

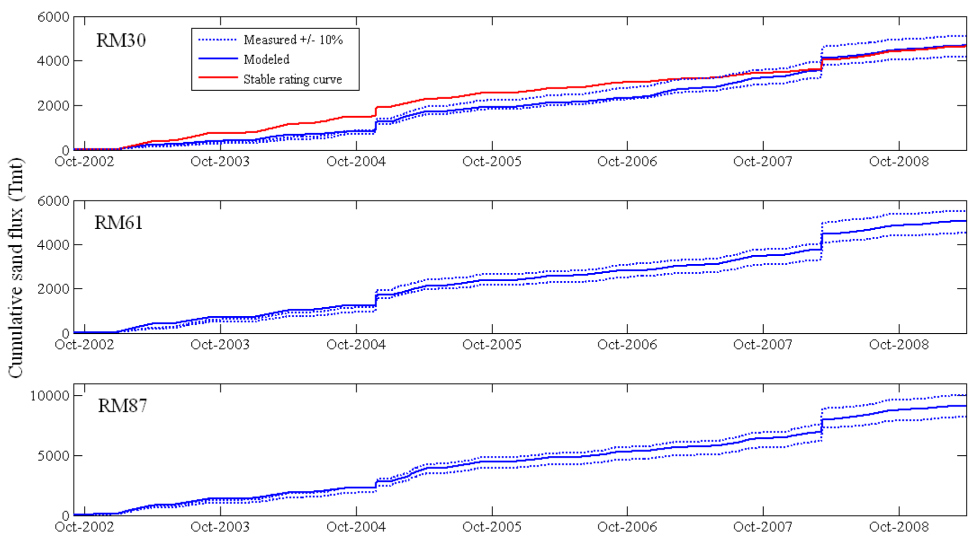


Figure 4 Comparison of measured (±10%) and modeled cumulative sand flux at the 3 gages.

Because the model was calibrated to match the total flux (Tmt in fig. 4 is thousand metric tons), the appropriate test of model performance is how well the model tracks the measurements within the time period, i.e. how well it simulates fining during tributary inputs and subsequent coarsening. We tested this by computing the percentage of time that the modeled cumulative flux was within the measurement uncertainty envelope. For each gaging site, the model was within the measurement uncertainty about 70% of the time with the greatest deviations occurring in the first two years of the simulation (thus potentially related to uncertainty in initial conditions). For comparison, the model was run for the UMC reach without computing changes in the bed composition, i.e. as a stable rating curve approach using the same calibration approach for *A*. The result (red line in fig. 4 top) illustrates the advantage of the size specific rating curve and bed composition calculations – the stable rating curve is within the measurement uncertainty only 30% of the time.

Figure 5 (left panel) compares modeled versus measured monthly sand fluxes for the three gages, an important comparison because Glen Canyon Dam operations are scheduled on a monthly basis and thus the model will likely be applied to evaluate different monthly operations. The model captures the general behavior fairly well, but there is also substantial variability and some indication that the model over-predicts medium-range fluxes and slightly under-predicts the highest fluxes. Defining *R* as the ratio of measured to modeled monthly flux allows for numerical comparisons as follows: median $R$ = {1.03, 1.30, 1.18}; % of months with $0.5<R<2$ = {96%, 86%, 96%}; and % of months with $0.8<R<1.25$ = {49%, 39%, 47%}, respectively for RM30, RM61, RM87 in each case. Thus, a very high percentage of the modeled fluxes are within a factor of 2 of the measurements but only about 50% are within a factor of 1.25.

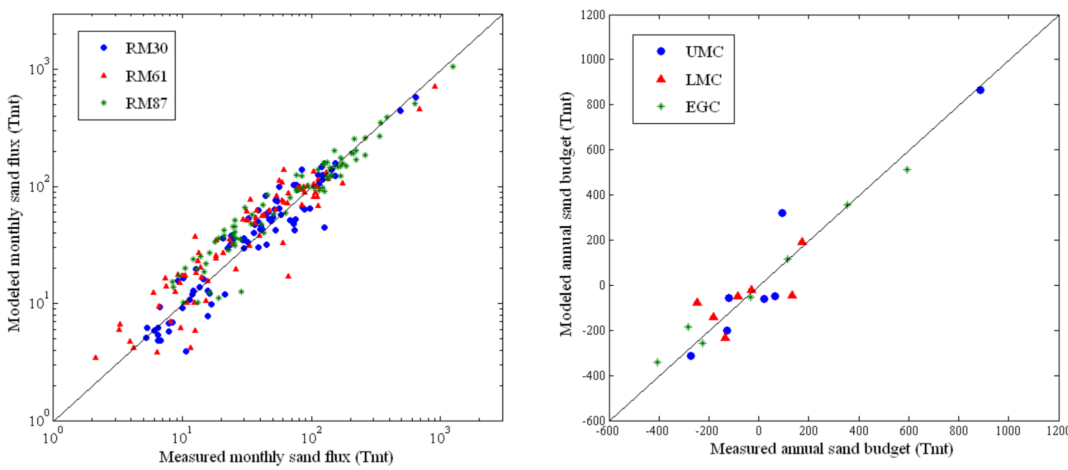


Figure 5 Comparison of measured and modeled monthly sand fluxes (left) and annual sand budgets (right) for the three monitoring sites and reaches.

Figure 5 (right panel) compares the measured and modeled annual sand budgets (i.e. annual change in storage on a mass basis) for each of the three reaches. The model proves capable of reproducing periods of substantial accumulation as well as erosion. The model performance for the sand budgets is a function of modeled annual sand fluxes and the ratios of measured to modeled annual flux are always within a factor of 1.5 and mostly within a factor of 1.25. Thus, model performance tends to improve as results are integrated over longer time scales.

The monthly and annual comparisons, in particular the comparison with a stable rating curve approach, indicate that the model is capable of simulating sequences of sand accumulation and bed fining followed by erosion and bed coarsening. This is further illustrated in fig. 6 which shows the modeled sand thickness (top) and median bed particle size (bottom) for the upper Marble Canyon reach. Several examples of accumulation and fining followed by erosion and coarsening are apparent, the most significant being the Paria River flooding in October 2006. Measurements of sand thickness and bed $D_{50}$ in short reaches (described previously) are in general agreement with the model for overall trends but the magnitudes can be quite different. The measurements exhibit greater variability likely because the reaches constitute only a small percentage of the modeling reaches (and may be located directly below tributaries).

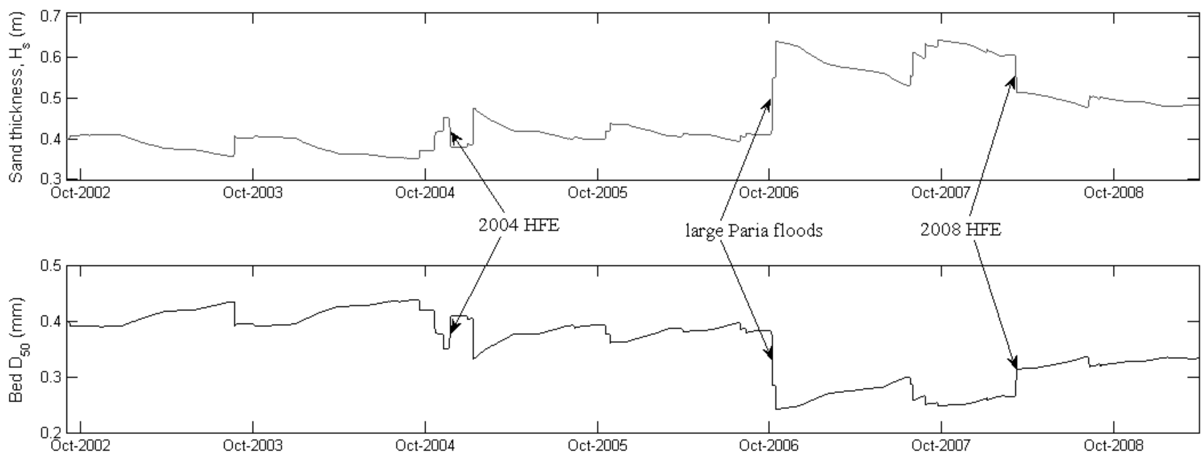


Figure 6 Modeled sand thickness (top) and bed $D_{50}$ (bottom), illustrating accumulation and fining during tributary floods and erosion and coarsening during mainstem high flows.

The HFEs in Nov-04 and Mar-08 provide excellent tests of the models' ability to simulate depletion of sand supply characterized by erosion and coarsening of the bed. Though these releases are designed to facilitate sandbar deposition in eddies, they also export significant amounts of sand from the system (Topping et al., 1999, 2006). Modeled and measured suspended-sand concentrations for the three gage sites for the two releases are shown in fig. 7; the supply depletion is apparent from the measured decrease in concentration during the constant peak discharge. In general, the model does a good job of simulating this decrease in concentration as well as differences between the two events in terms of the overall concentration magnitude. That is, the model predicts higher concentrations in Mar-08 (fig. 7 bottom row) than in Nov-04 (fig. 7 top row) and captures the downstream increase in concentration in Mar-08 that was less apparent in Nov-04. This indicates the ability of the model to accurately route the tributary sand inputs that control the antecedent reach-based supply for the events. There is a general tendency, however, for the model to under-predict concentrations on the rising limb of the hydrograph (and thus the peak concentration), particularly for the RM87 gage. This could be due to several of the various assumptions in the model (e.g. reach-averaging, complete bed mixing, lack of vertical structure in bed particle size, local equilibrium).

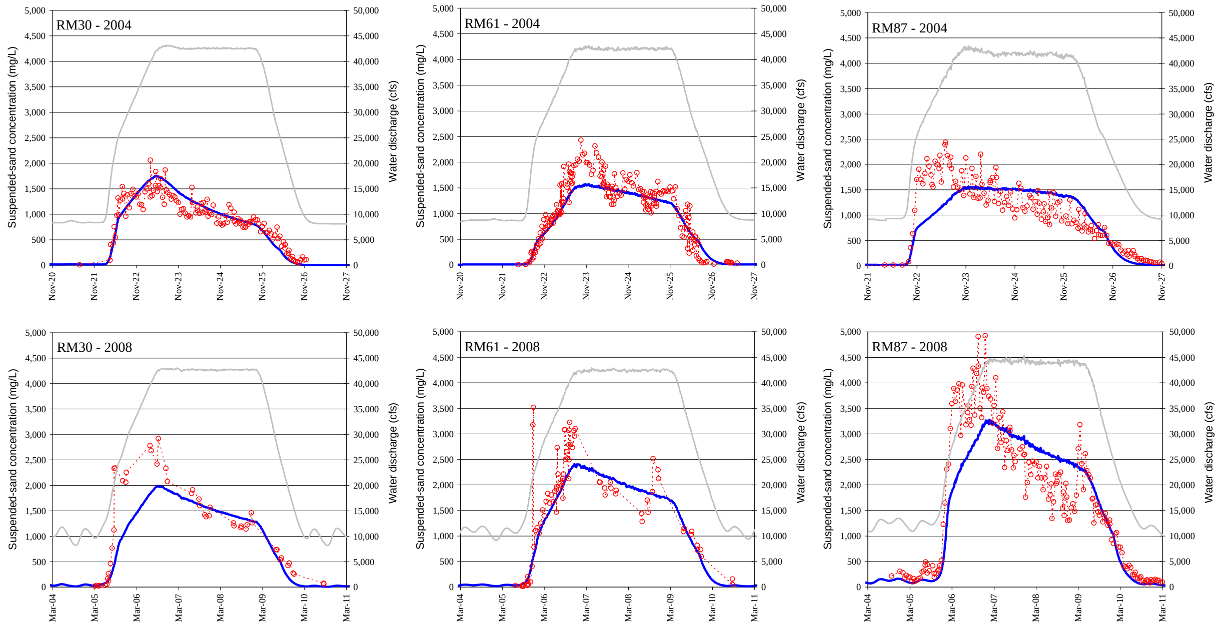


Figure 7 Measured (red) and modeled (blue) concentrations during the 2004 (top row) and 2008 (bottom row) HFEs at the 3 gages. Y-axis scale is the same on all panels for comparison.

## LIMITATIONS OF THE APPROACH

The modeling approach described herein is empirical in nature and substantially simplified with respect to the processes known to govern sediment transport in the study reach. The empiricism and simplifications were necessary to meet the goals of the modeling, in particular long-term simulation of the sand budget, but also result in several limitations as follows: 1) though the approach should have general applicability to rivers with a thin layer of sand overlying bedrock, the model coefficients (especially *A*) are site-specific and thus do not have general applicability; 2) the reach-averaged nature of the model requires that local effects of the pool-rapid-eddy morphology are not captured; 3) the use of long reaches and assumption of complete mixing of the bed sediment dictate that rapid responses due to changes in sediment supply will tend to be “smoothed out” in space and time, even though a short time step is necessary due to sub-daily discharge variation (i.e. hydropower fluctuations); 4) though it is known that the river bed is typically coarser in the deeper main channel than in shallower eddy environments and this gradation changes as a function of sand supply, the assumption of complete bed mixing precludes modeling this phenomenon; and 5) for the same reason, the model cannot simulate a scenario where a coarse surface layer temporarily precludes access to a finer substrate.

Finally, we note that the model was calibrated to conditions prevailing in the river between 2002 and 2009, and because of its empirical nature applications to conditions outside of the calibration period should be treated with extreme caution. For example, the model cannot simulate changes in the areal coverage of sand on the bed (e.g. Topping et al., 2007b); i.e. this effect is lumped into the calibration parameters (specifically *A*). Tests of the model against data from immediately post-dam (1965), when there was significantly more sand in the system, indicate that the model substantially under-predicts the concentrations. This could be due to changes in areal coverage or other effects (such as the vertical distribution of sand in the channel). These comparisons thus suggest that for conditions of very large accumulation (substantially more accumulation than occurred during 2002 – 2009) the model will tend to under-predict concentration and predict more accumulation than would actually occur. Because of this and other limitations, it is imperative that the model be used in concert with an ongoing monitoring program. This allows for continued evaluation of the model under different conditions, and improvements to the model formulation if necessary.

## CONCLUSIONS

The modeling approach described herein was designed to facilitate long-term simulations while capturing a fundamental mechanism controlling transport rates in the study reach, i.e. supply-driven changes in bed particle size and suspended-sediment concentration. This was achieved using a particle-size-specific form of sediment rating curves combined with a simplified form of the Exner equation for bed material size distribution accounting. A primary objective of the modeling approach was the ability to simulate long-term sand budgets for the Colorado River below Glen Canyon Dam. Model parameters were calibrated such that the total sand flux from each reach matched the measured flux for the period Sep-02 through Mar-09. Comparisons between measured and modeled monthly sand fluxes and annual sand budgets showed the model capable of simulating the variability in sand flux resulting from discharge variability as well as changes in sand supply due to tributary floods. Examination of modeled sand thickness and bed $D_{50}$ confirmed the models' ability to simulate accumulation of sand accompanied by bed fining during and immediately following tributary flooding, followed by erosion and bed coarsening during tributary quiescence and high flow releases from the dam. Comparisons of measured and modeled suspended-sand concentrations during the high flow releases in Nov-04 and Mar-08 indicate that the model can simulate the effects of different antecedent sand supply conditions as well as sand supply depletion during the peak discharge; however, the model tends to under-predict concentrations on the rising limb of the hydrograph most notably at the most downstream gage. The model was also shown to provide significant improvement over a stable rating curve approach for our study site, as expected. Finally, though these comparisons provide confidence for application of the model to forecast future conditions under various management scenarios, the empirical and simplified nature of the model lead to several limitations such that application of the model to conditions substantially outside those of the calibration period should be viewed with caution. Despite these limitations, the modeling approach provides an attractive alternative to the standard sediment rating curve approach for modeling long-term sediment budgets when the effects of supply limitation are important.

## REFERENCES

Grams, P.E., Schmidt, J.C., Topping, D.J., 2007. "The rate and pattern of bed incision and bank adjustment on the Colorado River in Glen Canyon downstream from Glen Canyon Dam, 1956–2000", GSA Bulletin, 119 (5/6), pp 556–575.

Hazel, J.E., Kaplinski, M., Parnell, R.A., Kohl, K., and Schmidt, J.C., 2008. "Monitoring Fine-Grained Sediment in the Colorado River Ecosystem, Arizona—Control Network and Conventional Survey Techniques", U.S. Geological Survey Open-File Report 2008-1276, 15 pp.

Kaplinski, M., Hazel, J.E., Jr., Parnell, R., Breedlove, M., and Schmidt, J.C., 2007. “Integrating topographic, bathymetric, and LIDAR surveys of the Colorado River in Grand Canyon to assess the effects of a flow experiment from Glen Canyon Dam on the Colorado River ecosystem”, Proceedings of the Hydrographic Society of America 2007 Annual Meeting, May 14−17, Norfolk, Va., 22 p.

Kaplinski, M., Hazel, J.E., Jr., Parnell, R., Breedlove, M., Kohl, K, and Gonzales, M., 2009. “Monitoring fine-sediment volume in the Colorado River ecosystem, Arizona: bathymetric survey techniques”, U.S. Geological Survey Open-File Report 2009–1207, 33 p.

Lovich, J.E., and Melis, T.S., 2005. “Lessons from 10 Years of Adaptive Management in Grand Canyon”, in The State of the Colorado River Ecosystem in Grand Canyon, edited by S.P. Gloss, J.E. Lovich, and T.S. Melis, U.S. Geological Survey Circular 1282, pp 207-220.

McLean, S.R., 1992. “On the calculation of suspended load for noncohesive sediments”, J. Geophys. Res. 97, 5759-5770.

Nilsson, C., Reidy, C.A., Dynesius, M., Revenga, C., 2005. “Fragmentation and flow regulation of the world’s large river systems”, Science, 308, 405-408.

Parker, G., Paola, C., and Leclair, S., 2000. “Probabilistic Exner sediment continuity equation for mixtures with no active layer”, J. Hyd. Eng., 126(11), 818-826.

Rahuel, J.L., Holly, F.M., Chollet J.P., Belleudy, P.J., Yang, G., 1989. “Modeling of riverbed evolution for bedload sediment mixtures”, J. Hyd. Eng., 115(11), 1521-1542.

Randle, T.J., and Pemberton, E.L., 1987. “Results and analysis of STARS modeling efforts of the Colorado River in Grand Canyon”, Glen Canyon Environmental Studies Report, 41 p. plus 95 figures and 26 tables.

Rubin, D.M., and Topping, D.J., 2001. “Quantifying the relative importance of flow regulation and grain-size regulation of suspended-sediment transport $\alpha$ and tracking changes in grain size of bed sediment $\beta$”, Wat. Resour. Res., 37(1), pp 133-146.

Rubin, D.M., Chezar, H., Harney, J.N., Topping, D.J., Melis, T.S., and Sherwood, C.R., 2007. “Underwater microscope for measuring spatial and temporal changes in bed-sediment grain size”, Sedimentary Geology, 202, pp 402–408

Schmidt, J.C., Topping, D.J., and Grams, P.E., 2004. “System-wide Changes in the Distribution of Fine-Grained Alluvium in the Colorado River Corridor Between Glen Canyon Dam and Bright Angel Creek, Arizona”, Final Report to the Grand Canyon Monitoring Research Center.

Syvitski, J.P.M, Vorosmarty, C.J., Kettner, A.J., and Green, P., 2005. “Impact of humans on the flux of terrestrial sediment to the global coastal ocean”, Science, 308, 376-380.

Topping, D.J., 1997. “Physics of flow, sediment transport, hydraulic geometry, and channel geomorphic adjustment during flash floods in an ephemeral river, the Paria River, Utah and Arizona”, University of Washington, Ph.D. thesis, 406 p.

Topping, D.J., Rubin, D.M., Nelson, J.M., Kinzel, III, P.J., and Bennett, J.P., 1999. “Linkage between grain-size evolution and sediment depletion during Colorado River floods”, in Webb, R.H., Schmidt, J.C., Marzolf, G.R., and Valdez, R.A., eds., The 1996 controlled flood in Grand Canyon: Washington, D.C., American Geophysical Union, Geophysical Monograph 110, pp 71-98.

Topping, D. J., Rubin, D.M. and Vierra Jr., L.E., 2000. “Colorado River sediment transport: Part 1: Natural sediment supply limitation and the influence of Glen Canyon Dam”, Wat. Resour. Res., 36(2), pp 515-542.

Topping, D.J., Schmidt, J.C., and Vierra, L.E., Jr., 2003. “Computation and Analysis of the Instantaneous-Discharge Record for the Colorado River at Lees Ferry, Arizona – May 8, 1921, through September 30, 2000”, U.S. Geological Survey Professional Paper 1677, 118 p.

Topping, D.J., Rubin, D.M., Schmidt, J.C., Hazel, J.E., Melis, T.S., Wright, S.A., and Kaplinski, M., 2006. “Comparison of sediment-transport and bar-response results from the 1996 and 2004 controlled flood experiments on the Colorado River in Grand Canyon”, Proceedings, 8th Federal Interagency Sedimentation Conference, April 2-6, 2006, Reno, NV, CD-ROM only, ISBN 0-9779007-1-1.

Topping, D.J., Wright, S.A., Melis, T.S., and Rubin, D.M., 2007a. “High-resolution measurements of suspended-sediment concentration and grain size in the Colorado River in Grand Canyon using a multi-frequency acoustic system”, in Proceedings of the 10th International Symposium on River Sedimentation. August 1–4, Moscow, Russia. Volume III.

Topping, D.J., Rubin, D.M, and Melis, T.S., 2007b. “Coupled changes in sand grain size and sand transport driven by changes in the upstream supply of sand in the Colorado River: Relative importance of changes in bed-sand grain size and bed-sand area”, Sedimentary Geology 202, pp 538–561.

Wiele, S.M., Graf, J.B., and Smith, J.D., 1996. “Sand deposition in the Colorado River in the Grand Canyon from flooding of the Little Colorado River”, Water Resources Research, 32(12), pp 3579-3596.

Wiele, S.M., Andrews, E.D., and Griffin, E.R., 1999. “The effect of sand concentration on depositional rate, magnitude, and location in the Colorado River below the Little Colorado River”, in The Controlled Flood in Grand Canyon, Geophysical Monograph 110, edited by R.H. Webb, J.C. Schmidt, G.R. Marzolf, and R.A. Valdez, pp 131-145.

Wiele, S. M., P. R. Wilcock, and P. E. Grams, 2007. “Reach-averaged sediment routing model of a canyon river”, Water Resour. Res., 43, W02425, doi:10.1029/2005WR004824.

Wright, S.A., Melis, T.S., Topping, D.J., and Rubin, D.M., 2005. “Influence of Glen Canyon Dam Operations on Downstream Sand Resources of the Colorado River in Grand Canyon”, in The State of the Colorado River Ecosystem in Grand Canyon, edited by S.P. Gloss, J.E. Lovich, and T.S. Melis, U.S. Geological Survey Circular 1282, pp. 17-31.

Wright, S.A., Schmidt, J.C., Melis, T.S., Topping, D.J. and Rubin, D.M. 2008. “Is there enough sand? Evaluating the fate of Grand Canyon sandbars”. GSA Today, 18 (8).