BANKFULL MOBILE PARTICLE SIZE AND ITS PREDICTION FROM A SHIELDS-TYPE APPROACH

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Abstract Critical dimensionless shear stress values τ^*_c (Shields 1936) (also called Shields values) are often applied to solve a wide range of incipient motion questions in natural streams. A common application for a Shields curve is the computation of the bankfull mobile particle size. However, the original Shields values were obtained from experiments conducted in a narrow set of flume conditions and often fail to produce accurate incipient motion results in natural streams. This study examined how the computed bankfull mobile particle size (D_{cbf}) depends on the specific Shields value used to compute it and how D_{cbf} values and their relations to the bed surface D_{50} size vary over a wide range of stream types. Four Shields-type curves and their predictions of D_{cbf} were compared: 1) the **original** Shields curve τ^*_c, 2) a **modified** Shields curve $\tau^*_{c\ mod}$ (Parker et al. 2003), 3) a **bankfull** Shields curve τ^*_{bf} computed for bankfull flow and the bed D_{50} size over a wide range of model streams and 4) a **critical bankfull** Shields curve τ^*_{cbf} computed from bankfull flow and the largest bankfull mobile bedload particle size ($D_{max\ bf}$) observed at 13 sites in mountain streams. The values of τ^*_c, τ^*_{bf} , and τ^*_{cbf} are similar for mobile gravel-bed streams, but differ widely for silt- and sand beds as well as for steep plane-bed and step-pool mountain streams. Critical bankfull Shields values τ^*_{cbf} increased steeply with gradient and bed coarseness, ranging within 0.03 – 0.085 for mixed plane-bed/pool-riffle coarse gravel-bed streams, within 0.05 to 0.36 for plane-bed streams that have occasional pool-riffle sequences forced by sharp channel bends, and within 0.17 – 0.67 for cobble-bed step-pool streams. Using the original or modified Shields curve with τ^*_c of 0.03 – 0.06 to predict the bankfull mobile particle size in coarse-grained streams can be appropriate in mobile gravel- and cobble-bed streams with high sediment supply (such as alternate bars, braided, pool-riffle morphology as well as in torrents). However, compared to the measured bankfull mobile particle size $D_{max\ bf}$, a Shields value of 0.03 – 0.06 overpredicts D_{cbf} by a factor of approximately 5 in steep and poorly mobile plane-bed streams and by a factor of approximately 10 for step-pool streams. Shields values to compute the bankfull mobile particle size need to be carefully chosen.

INTRODUCTION

Definition and background of Shields values Shields (1936) quantified the critical dimensionless shear stress τ^*_c (also called Shields values) in terms of the flow hydraulics (i.e., the product of slope and flow depth) at which particles from a bed with a mean particle size D_m start to become entrained. The details of how incipient motion was determined remain debated (Kennedy 1995; Buffington 1999, 2000; Garcia 2000). τ^*_c is defined as

$$\tau^*_c = \frac{\tau_c}{(\rho_s - \rho_f) \cdot g \cdot D_{50}} = \frac{\rho_f \cdot g \cdot R_c \cdot S}{(\rho_s - \rho_f) \cdot g \cdot D_{50}} \quad \text{(1a and b)}$$

where ρ_f and ρ_s = water and particle densities, g = acceleration due to gravity, R_c = hydraulic radius at which incipient motion occurs, S = bed gradient, and D_{50} = median bed surface particle size. Shields (1936) specified the bed material size by its bulk mean size D_m which for the mostly well-sorted and unarmored particle-size distributions in his experiments can be expected to be similar to the bed surface D_{50} size. He also planed the bed before each run, and bedforms were absent when particles started to move.

The Shields curve presents τ^*_c as a function of the dimensionless Reynolds particle number Re_p

$$Re_p = \frac{\sqrt{g \cdot R \cdot S} \cdot D_{50}}{\nu} \quad (2)$$

where ν = kinematic viscosity (1.3E-6 and 1.55E-6 m^2/s for water temperatures of 5 and 12°C). Being mostly dependent on D_{50}, Re_p increases with D_{50}. For beds of silt and sand, the user first computes Re_p and then finds the associated value of τ^*_c on the curve. For gravel beds ($Re_p > \approx 300$), the Shields curve provides a constant value of

τ^*_c = 0.056 (Rouse 1939) that was subsequently lowered to 0.044 or 0.03 depending on which rendition of the Shields curve is consulted (Paintal 1971; Miller et al. 1977; Yalin and Karahan 1979; Brownlie 1981; Buffington and Montgomery 1997; or Parker et al. 2003). The constancy of τ^*_c for Re_p >300 makes the Shields equation easy to use in coarse-grained streams that typically have high Re_p values. A user selects a value for τ^*_c (between 0.03 and 0.06) and solves the Shields equation (Eq. 1) for either critical flow (R_c) given a specified D_{50} or for critical particle size (D_c) given a specified flow (Eq. 3 a and b).

$$R_c = \frac{(\rho_s - \rho_f) \cdot g}{\rho_f \cdot g} \cdot \frac{\tau^*_c \cdot S}{D_{50}} \quad \text{or} \quad D_c = \frac{\rho_f \cdot g}{(\rho_s - \rho_f) \cdot g} \cdot \frac{R \cdot S}{\tau^*_c} \qquad \text{(3a and b)}$$

The mathematical ease of solving the Shields equation for either R_c or D_c make it tempting to use the Shields equations as a general purpose tool for all sorts of incipient motion questions. Consequently, τ^*_c values are often applied to solve a wide range of incipient motion questions in natural streams.

Problems with using Shields values for predicting the bankfull mobile particle size A common application of the Shields curve is the computation of the bed particle size that can be entrained at bankfull flow in natural streams (e.g., Olsen et al. 1997; Buffington and Montgomery 1999a; Buffington et al. 2004; Kaufmann et al. 2008). However, by definition of the Shields value, solving the Shields equation for a critical particle size D_c and using bankfull flow is based on the inherent assumption that bankfull flow entrains the bed D_{50} size. As a consequence, the Shields curve τ^*_c is only valid in streams that transport their bed surface D_{50} size at bankfull flow. The assumption of bankfull mobility of the bed surface D_{50} size is not generally valid, but holds true only under specific circumstances. In sand- and fine gravel-bed streams, bankfull flow entrains particle sizes close to the surface D_{max} size, i.e., particles larger than D_{50}. Bankfull flow entrains the D_{50} size in some gravel- and cobble-bed streams but not in others. Variability occurs because particle mobility is not determined only by the general stream coarseness and steepness, but also by the amount of sediment supplied to a stream (Figure 1). Torrents receive a high supply of sediment of all sizes and have beds of low structural stability; here bankfull flow likely entrains particle sizes close to the bed D_{50} size. By contrast, step-pool mountain streams receive low sediment supply, and many of the larger particles have high structural bed stability; here, bankfull flows entrains particle size much finer than the bed surface D_{50} size.

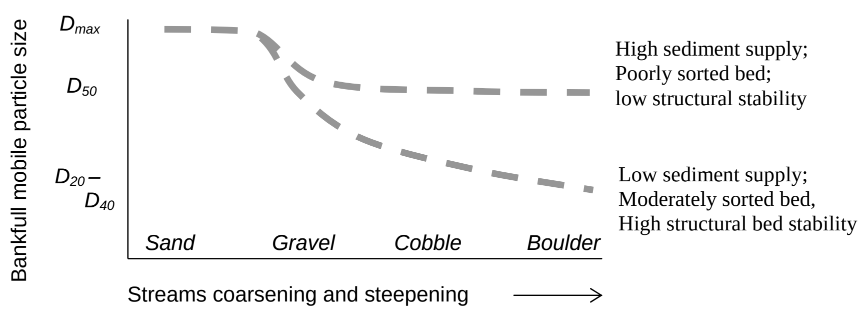

Figure 1 Hypothetical relationships between bankfull mobile particle size and stream steepness and bed coarseness.

Predicting the bankfull entrainable particle size requires the critical bankfull Shields value τ^*_{cbf} To accurately predict the bed material particle size mobile at bankfull flow D_{cbf}, a Shields-type curve is needed that is derived from bedload particle sizes measured to be mobile at bankfull in different streams (τ^*_{cbf}). Yet, τ^*_{cbf} curves based on bankfull flow and the average largest bedload particle size observed to be entrainable at bankfull flow has so far not been computed for a wide range of natural streams. This study provides τ^*_{cbf} values for several coarse mountain streams.

Study objectives To show differences among various Shields-type curves and how those differences affect the computed bankfull mobile particle size, this study compares four Shields curves: 1) the **original** Shields curve $\tau^*_{c\ orig}$ fitted by Rouse (1939), 2) a Shields curve $\tau^*_{c\ mod}$ **modified** by Parker et al. (2003), 3) Shields values computed for **bankfull flow** τ^*_{bf}, and 4) **critical bankfull** Shields values τ^*_{cbf}. The first three Shield values are computed for 11 model (i.e., hypothetical) streams that span a wide range stream types. The 4th, the critical bankfull Shields value τ^*_{cbf} is computed from the bankfull mobile particle size $D_{max\ bf}$ measured at 13 sites in coarse mountain streams. The study then makes comparisons among the computed bankfull mobile particle sizes D_{cbf} and with the observed $D_{max\ bf}$.

METHODS

Selection of model streams **Selected stream types** In order to compute the original and modified Shields curves $\tau^*_{c\ orig}$ and $\tau^*_{c\ mod}$ as well as the bankfull Shields curve τ^*_{bf} over a wide range of stream types and Reynolds numbers Re_p (Eq. 2), a series of 11 hypothetical model streams was envisioned that cover Re_p between 0.4 and 81,000. Each modeled stream represents a specific stream type, and together, the 11 model streams cover a range from low gradient, silt- and sand-bed dune-ripple valley streams to gravel-bed pool-riffle and plane-bed streams, to headwater step-pool streams and finally cascades (Montgomery and Buffington 1997). Within the large category of mobile gravel-bed streams, this study distinguished between pool-riffle and alternate bar morphology, the latter more fine-grained and with a longer spacing of pools. Coarse gravel plane-bed, cobble step-pool streams, and cascades are representatives of relatively low sediment supply; a braided gravel-bed stream and a steep cobble-bed torrent represent coarse-grained streams with high sediment supply. Torrents receive a high sediment supply from a wide spectrum of particle sizes which causes a loose, poorly structured bed in which most bed particles are mobile during commonly occurring highflows.

Numerical values of the selected channel parameters To compare roughly similar-sized model streams, bankfull depths were selected as wadeable at low flow with the exception of the low-gradient, fine-grained streams where bankfull flow was allowed to exceed wadeability. Within these constraints, numerical values were selected for the bed surface D_{50} size, stream gradient S, and bankfull hydraulic radius R_{bf} of each model stream type. The selected values were based on the Montgomery and Buffington (1997) stream classification as well as on the authors' field experience in Rocky Mountain streams. The focus here is not on an individual stream type; the point is to show how Shields values, and thus D_{cbf} and D_{50}/D_{cbf}, vary over a wide range of stream types from headwaters to valley streams.

The bed material particle size D_{50} was set to 0.01 mm for silt-bed streams and increased steadily to reach 150 mm in cascade, thus covering 4 orders of magnitude over the model streams (Table 1). Stream gradient S was set to 0.0002 m/m for a silt-bed stream and to 0.3 m/m for a cascade, increasing for each listed stream type and covering more than three orders of magnitude. The bankfull hydraulic radius R_{bf} was assumed to decrease with stream gradient and was set to start at 1.5 m in a silt-bed dune-ripple stream and decreased to 0.25 m in a cobble-bed cascade.

Table 1 Channel characteristics possibly encountered in a series of model streams that are assumed wadeable at low flow but may exceed wadeablity at bankfull flow. Also computed are Reynolds particle numbers Re_p and R_p, critical Shields values τ^*_c , bankfull Shields values τ^*_{bf}, and the computed bankfull mobile particle size D_{cbf}.

General Bed material	Stream Type (*in sensu* Montgomery & Buffington 1997)	Surf. Part. size D_{50} (mm)	Stream gradient S (m/m)	Bankfull		Reynolds particle number		Critical Shields value		Bankf. Shields Value $\tau^*_{bf} = f(R_{bf})$ (-)	D_{cbf} (mm) =	
				Hydr. radius R_{bf} (m)	Shear stress τ_{bf} (Pa)	Re_p (-)	R_p (-)	Shields (1936) $\tau^*_c = f(Re_p)$ (-)	Parker et al. ('03) $\tau^*_c = f(R_p)$ (-)		$f(\tau^*_{c\ orig})$	$f(\tau^*_{c\ mod})$
Silt	**D**une-**r**ipple	0.01	0.0002	1.50	2.9	0.42	0.10	0.280[#]	0.444	18.1	0.65	0.4
Fine sand	**D**une-**r**ipple	0.12	0.0005	1.10	5.9	6.8	4.07	0.035	0.047	2.77	9.5	7.0
Coarse sand	**D**une-**r**ipple	1	0.0008	0.90	7.1	65	98	0.042	0.017	0.43	10	26
Sand-gravel	**A**lternate **b**ars	5	0.0015	0.75	11	404	1094	0.055	0.025	0.14	12	28
Med. gravel	**A**lternate **b**ars	15	0.003	0.50	15	1400	5686	Not defined by Shields (1936) but commonly taken as 0.056	0.028	0.060	16	33
Med. gravel	**Br**aided	19	0.005	0.35	17	1915	8105		0.028	0.056	19	37
Coarse gravel	**P**ool-**r**iffle	40	0.006	0.50	39	6095	24758		0.029	0.060	43	83
Coarse gravel	**P**lane-**b**ed	60	0.025	0.35	86	13522	45484		0.029	0.088	94	180
Cobble	**Tor**rent*	90	0.05	0.20	98	21684	83560		0.030	0.067	108	204
Cobble	**S**tep-**p**ool	110	0.09	0.25	221	39753	112907		0.030	0.124	243	459
Cobble	**Cas**cade	150	0.20	0.25	491	80810	179792		0.030	0.201	539	1017

* steep mountain stream with high sediment supply in which large particles are mobile at common high flows;
[#] extrapolated value; Letters in bold are used as abbreviations for stream types in Figures 6 and 7.

The assigned channel parameters were checked against published data from natural streams. The bankfull shear stress τ_{bf} (= $\rho_f \cdot g \cdot R_{bf} \cdot S$) increased steadily from 2.9 for the silt-bed dune-ripple stream to 491 Pa for the cobble cascade. The D_{50} sizes selected for the 11 model streams fall within the relationship of D_{50} to τ_{bf} shown by Buffington and Montgomery (1999) and Buffington et al. (2004) for numerous stream measurements and confirm that the model streams are representative of natural streams. To sketch the effect that energy dissipation due to grain and form roughness might have on the effective hydraulic radius, an estimated roughness-correction factor of $^1/_3$ was applied to the bankfull hydraulic radius of all model streams to yield $R_{fb\ cor} = {}^1/_3\ R_{bf}$. A factor of $^1/_3$ is within the range of roughness corrections of ¼ to ½ applied by Kaufmann et al. (2008).

Computation of Shields curves from the 11 model streams The channel characteristics assigned to each of the model streams (D_{50}, S, and the bankfull R_{bf}) were used to compute the original Shields curve, the modified Shields curve, and the bankfull Shields curve (Table 1).

Original Shields curve $\tau^*_{c\ orig}$ The original Shields curve for bankfull flow was computed by first calculating the Reynolds particle number Re_p from the assigned values of D_{50}, S and R_{bf} each model stream. The values of $\tau^*_{c\ orig}$ were then read off the Rouse (1939) graph for each Re_p. The original Shields experiments covered a relatively narrow range of Re_p between 2 to 500 and do not provide τ^*_c values for Re_p of 10,000 – 100,000 that are obtained in natural coarse gravel- and cobble-bed streams. A value of 0.056 was assumed for $Re_p > 500$.

Modified Shields curve $\tau^*_{c\ mod}$ Brownlie (1981) devised an analytical description of the Shields curve that enables computation of τ^*_c as a function of R_p ($\tau^*_c = 0.22\ R_p^{-0.6} + 0.06 \cdot 10^{(-7.7Rp\wedge-0.60)}$). R_p is a modification of the Reynolds particle number that substitutes the R and S product from the square root term in Eq. 2 by a density and D_{50} product. This modification makes R_p almost entirely dependent on D_{50}.

$$R_p = \sqrt{g \cdot \left(\frac{\rho_s\text{-}\rho_f}{\rho_f}\right) \cdot D_{50}} \cdot \frac{D_{50}}{\nu} \qquad (4)$$

The numerical values of Re_p and R_p computed for the same sets of S, D_{50} and R of the model streams were identical only at $Re_p \approx 11$. At a low Re_p of 0.4, Re_p is 4 times larger than R_p; at a high Re_p of 2000, R_p is about 4 times larger than Re_p. For $Re_p > 2000$, the ratio Re_p/R_p settles near 0.3. Parker et al. (2003) adopted the Brownlie modification but multiplied τ^*_c by 0.5 to accommodate incipient motion results by Neill (1968). His τ^*_c values measured on very well to poorly sorted beds of coal particles with D_{50} sizes of 3 to 20 mm mostly ranged between 0.03 and 0.06. The modified "lowered" Shields curve (Parker et al. 2003) takes a constant value of $\tau^*_{c\ mod} = 0.03$ for high values of R_p where the curve is flat.

The modified Shields curves by Brownlie (1981) and Parker et al. (2003) were computed for the 11 model streams by first calculating the Reynolds particle number R_p from the assigned values of D_{50}, S and R_{bf} and using R_p to compute τ^*_c from the analytical expression $\tau^*_c = f(R_p)$. All three curves are plotted in Figure 2. Since Re_p and R_p differ somewhat, the Brownlie (1981) modification is off-set to the side of the original Shields curve, while the Parker et al. (2003) modification is off-set to the side and lower than the original curve.

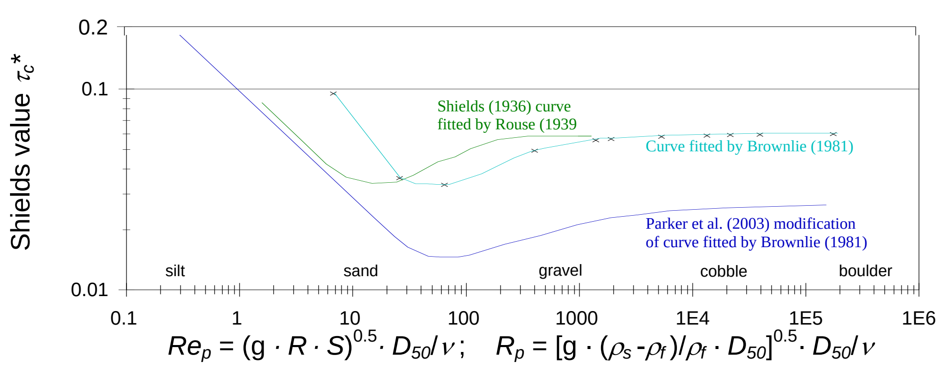

Figure 2 Original Shields curve $\tau^*_{c\ orig}$ (Shields 1936) as fitted by Rouse (1939), the Brownlie (1981) modification (using R_p instead of Re_p), and the modified curve $\tau^*_{c\ mod}$ by Parker et al. (2003).

This study computed the bankfull mobile particle size D_{cbf} with $\tau^*_{c\ orig}$ = 0.056 and $\tau^*_{c\ mod}$ = 0.03 in gravel and cobble-bed model streams specifically to show how D_{cbf} is affected by the choice of τ^*, because Shields values within 0.03 to 0.056 are often used for a wide range of mountain streams (e.g., Olsen et al. 1997; Lorang and Hauer 2003; Conesa-Garcia 2007, Kaufmann et al. 2008), even though critical Shields values higher than 0.03 and 0.056 for rough and steep streams have been repeatedly reported (e.g., Bathurst et al. 1983, 1987; Lepp et al. 1993; Rosgen 1996; Buffington and Montgomery 1997; Shvidchenko and Pender 2000; Buffington et al. 2004; Mueller et al. 2005; Lamb et al. 2008; Recking 2009).

Shields curve for bankfull flow τ^*_{bf} and $\tau^*_{bf\ cor}$ Shields values for bankfull flow τ^*_{bf} were computed by inserting the values of the channel characteristics assigned to each model stream D_{50}, S, and R_{bf} into the Shields equation (Eq. 1) (Table 1). A roughness corrected $\tau^*_{bf\ cor} = {}^1/_3\ \tau^*_{bf}$ was computed using the estimated roughness corrected $R_{bf\ cor}$.

Computations of critical bankfull Shields values (τ^*_{cbf}) from mountain study streams To compute the critical dimensionless shear stress for bankfull flow τ^*_{cbf}, accurate measurements of the coarsest bedload particles are needed. τ^*_{cbf} could not be computed for the 11 model streams because data on the particle size entrained at bankfull flow are not available. Instead, τ^*_{cbf} was computed for 13 study sites in mostly Rocky Mountain gravel- and cobble-bed streams where bedload transport was measured during snowmelt runoff using bedload traps.

Field measurements The study sites encompassed three general stream types: I) mixed plane-bed and pool-riffle streams, II) steep plane-bed streams with occasional pool-riffle sequences forced by sharp bends in the typically incised channels, and III) step-pool/cascade streams. Stream gradients ranged from 0.012 to 0.093, surface D_{50} sizes from 49 to 108 mm, and bankfull hydraulic radii from 0.20 – 0.57 m (Table 2; see Table 1 in Potyondy et al., this volume for further description of site characteristics).

Table 2 Channel characteristics, Shields values $\tau^*_{c\ orig}$ and $\tau^*_{c\ mod}$, bankfull Shields values τ^*_{bf}, and critical bankfull Shields values τ^*_{cbf} computed for the mountain study streams.

Stream	S (m/m)	R_{bf} (m)	D_{50} (mm)	Re_p (-)	$\tau^*_{c\ orig}$	$\tau^*_{c\ mod}$	$\tau^*_{bf} = f(R_{bf}, S, D_{50})$	$D_{max,bf}$ (mm)	$D_{50}/D_{max,bf}$	$\tau^*_{cbf} = f(R_{bf}, S, D_{max.bf})$	$D_{cbf} = f(\tau^*_{c\ orig})$ (mm)	Dominant stream morphology
Litl. Granite '02	0.012	0.23	67	8525	0.056	0.028 – 0.029	0.046	32	2.1	0.053	30	Mixed plane-bed and pool-riffle
Halfmoon, riffle '04	0.014	0.57	49	10547			0.097	56	0.9	0.085	185	
Halfmoon, bar '04	0.014	0.52	49	10114			0.089	127	0.4	0.035	79	
Oak (Milhous '73)	0.014	0.27	50	7393			0.045	65	0.8	0.035	40	
Squaw '88 (Bunte '96)	0.021	0.32	42	8332			0.095	101	0.4	0.040	72	
E. Dallas '07	0.017	0.32	58	10301			0.056	61	0.9	0.053	58	Plane-bed with forced pool-riffle sequences
Litl. Granite '99	0.017	0.38	59	11425			0.066	51	1.2	0.077	70	
St. Louis '98	0.017	0.34	76	13947			0.025	36	2.1	0.097	63	
Cherry '99	0.025	0.39	49	12130			0.115	16	3.1	0.363	105	
Hayden '05	0.038	0.25	63	14918			0.093	35	1.8	0.174	104	Step-pool
Fool '09	0.044	0.20	52	10018			0.074	16	3.3	0.240	69	
E. St. Louis '01	0.093	0.26	108	40331			0.134	22	4.9	0.673	259	
E. St. Louis '03	0.093	0.26	108	40331			0.134	25	4.3	0.571	259	

Bedload traps used in the study streams consist of an aluminum frame 0.3 by 0.2 m in size. Gravel bedload is collected in an attached net 0.9 – 1.6 m long and with a mesh width just below 4 mm. Bedload traps are mounted onto ground plates 0.43 by 0.37 m in size that are anchored on the stream bottom with metal stakes (**Figure 3**). Bedload traps can collect the largest mobile bedload particles quite well, due to their large opening size, large net capacity, low propensity for net clogging, long deployment times (30 - 60 min) that average over short-term fluctuations, and avoidance of direct ground contact (Bunte et al. 2004, 2007, 2008; Potyondy et al. this volume). Because bedload traps and their deployment are designed for minimizing known shortcoming in measuring gravel and cobble bedload transport in mountain streams, their data are believed to provide fairly accurate relationships between the average largest bedload particle size (D_{max}) and discharge (Q) (= flow competence curves) (Figure 4).

Figure 3 Bedload traps deployed in two of the study streams at the beginning of the highflow season.

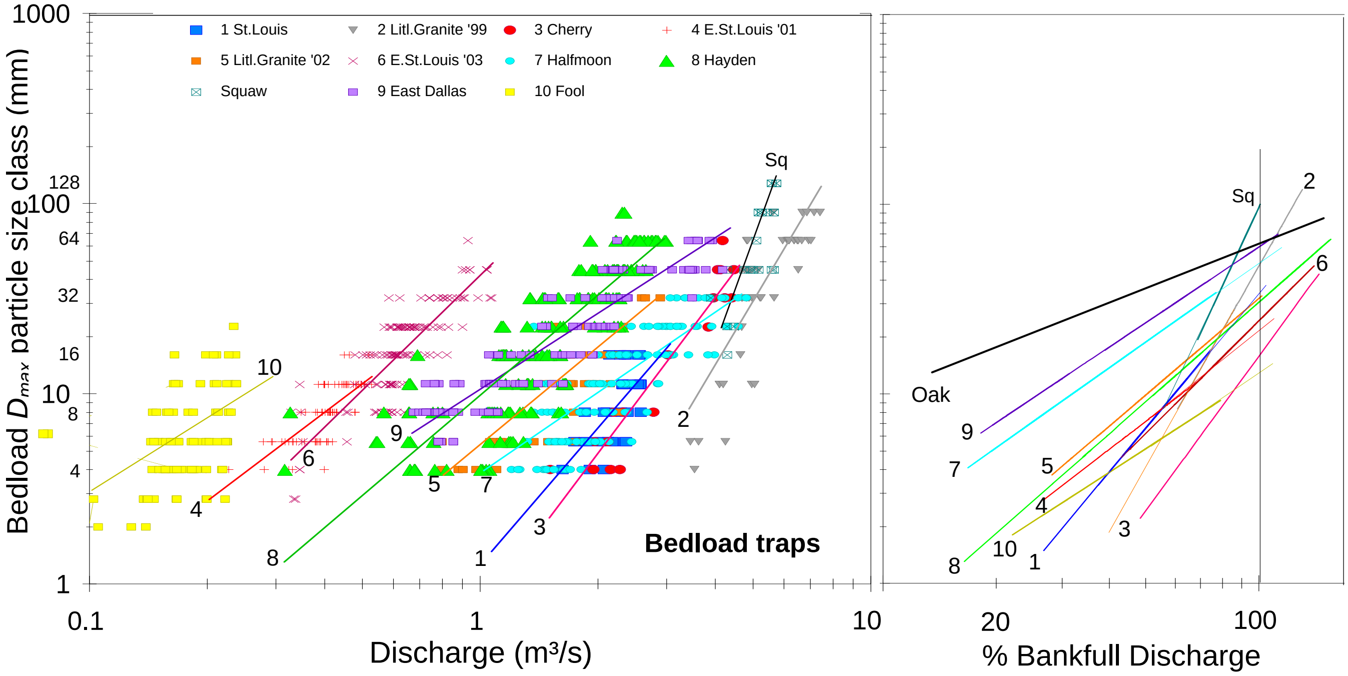

Figure 4 Largest bedload particle size (D_{max}) from each sample plotted versus discharge and fitted flow competence curves for ten mountain streams where bedload was collected using bedload traps (left); Flow competence curves as functions of the percent of bankfull flow (right).

Flow competence curves and computation of the bankfull mobile particle size For each of the study streams, the relationship between the largest measured bedload particle size class per sample (D_{max}) and discharge (Q) at the time of sampling was described by power functions fitted to log-transformed values of D_{max} and Q (Figure 4, left). The power functions provide flow competence curves in the general form

$$D_{max} = a\, Q^{b} \quad (5)$$

where a is a regression coefficient and b is the exponent. The flow competence curves were then expressed in terms of bankfull discharge where c is a regression coefficient (Figure 4, right).

$$D_{max} = c \cdot \%Q_{bf}{}^{b} \quad (6)$$

Flow competence relationships from Squaw Cr. where gravel bedload was measured with a large net-frame sampler (Bunte 1996) and from Oak Cr. (OR Coast Range) where bedload was collected with a vortex sampler (Milhous 1973) were added to these data sets. The bankfull mobile particle size $D_{max,bf}$ was taken as the value predicted by the flow competence curves (Eq. 6) for bankfull flow (Figure 4, right). To account for the inherent underprediction of estimates from power functions, $D_{max,bf}$ was multiplied by the Ferguson (1986, 1987) bias correction factor CF_{Ferg} = exp (2.651 s_y^2) where s_y is the standard error of the y-estimate. Since the flow competence relationships were relatively well defined, CF_{Ferg} values for our study streams were small (1.03 – 1.3) with a mean of 1.09.

$$D_{max,bf} = CF_{Ferg} \cdot a\, Q_{bf}^{\ b} \tag{7}$$

Values of $D_{max\ bf}$ are listed in Table 2 for all study streams. For streams with mixed plane-bed and pool-riffle morphology, ratios of $D_{50}/D_{max\ bf}$ are 0.4 – 2.1. For plane-bed streams with forced pool-riffle sequences, $D_{50}/D_{max\ bf}$ ratios are 0.9 – 3.1, meaning that the bankfull mobile particle size is about 1 to $^1/_3$ of the bed D_{50} size. Step-pool streams have ratios of 1.8 – 4.9, meaning that the bankfull mobile particle size is about ½ to $^1/_5$ of the bed D_{50} size.

Backcalculation of critical Shields values τ^*_{cbf} Critical bankfull Shield values τ^*_{cbf} were back-calculated from S, R_{bf} and the bankfull mobile particle $D_{max,bf}$ measured in all ten study streams and by solving the Shields equation (Eq. 1) for τ^*_{cbf} (Table 2). Values of ρ_f and ρ_s were taken as 1,000 and 2,650 kg/m^3, respectively.

RESULTS AND DISCUSSION

Comparison of the four Shields type curves For graphical comparison, the four Shields curves $\tau^*_{c\ orig}$, $\tau^*_{c\ mod}$, τ^*_{bf}, and τ^*_{cbf} are plotted together as functions of Re_p or R_p respectively (Figure 5). The $\tau^*_{c\ orig}$, $\tau^*_{c\ mod}$, and τ^*_{bf} curves are computed from the channel characteristics of the 11 model streams that represent a wide range of Reynolds numbers and stream types (see acronyms next to data points). The critical bankfull Shields values τ^*_{cbf} that were computed from flow competence measured at the 13 mountain stream study sites encompass only a few stream types and therefore a narrow range of Reynolds particle numbers.

Variability of τ^*_{bf} over model streams and comparison with τ^*_c Bankfull Shields values τ^*_{bf} are shown by the gray band in Figure 5. The top side of the band represents τ^*_{bf} values computed from R_{bf}; the bottom side are $\tau^*_{bf\ cor}$ values plotted to sketch effects of using a roughness-corrected $R_{bf\ cor}$. $\tau^*_{bf\ cor} = {}^1/_3\tau^*_{bf}$ because $R_{bf\ cor}$ was set to $^1/_3 R_{bf}$.

Similar to the original Shields curve, the curve for τ^*_{bf} varies non-monotonically with Re_p, i.e., bed material size and stream gradient. For low Re_p numbers, τ^*_{bf} is near 20 in silt-, and near 1 in sand-bed streams. These values exceed τ^*_c by about an order of magnitude and correspond well to those offered by Dale and Friend (1998), Garcia (2000), Church (2002) and Parker et al. (2003) for fine-grained streams. The moderately coarse and mobile gravel-bed model streams (alternate bars, braided, and pool-riffle) as well as the mobile mountain torrent have the lowest bankfull Shields values of τ^*_{bf} of near 0.06, similar to the original Shields curve. From this minimum, bankfull Shields values τ^*_{bf} increase with stream gradient and stream coarseness to reach values of 0.09, 0.12, and 0.2 for steep gradient plane-bed, cobble-bed step-pool streams, and the cascade, respectively. The τ^*_{bf} values computed for the model pool-riffle, plane-bed and step-pool streams fall within the relationships of τ^*_{bf} *vs.* bankfull shear stress τ_{bf} shown for measured streams (Buffington and Montgomery 2001; Buffington et al. 2004). The bankfull Shields values τ^*_{bf} for the mountain model streams (pool-riffle to cascade) increase with stream gradient and coarseness and are similar to τ^*_{bf} values indicated by Mueller et al. (2006) and to τ^*_{ci} values by Rosgen (1996).

Variability of τ^*_{cbf} over the study streams and comparison with τ^*_c The values of τ^*_{cbf} that were back-calculated from captured bankfull bedload particle sizes in coarse gravel and cobble mountain study streams show a very steep increase with stream gradient and bed coarseness, increasing from near 0.035 to 0.67, an almost 20 fold range. The most mobile of the study streams (mixed plane-bed and pool-riffle morphology) with gradients of 0.014 - 0.021 had values of τ^*_{cbf} of 0.03 – 0.085 that fall within the range predicted by the original and the modified Shields curves τ^*_c. For plane-bed streams with less sediment supply, τ^*_{cbf} values ranged between 0.053 and 0.36, up to 6 times higher than the original Shields curve $\tau^*_{c\ orig}$ and up to 10 times higher than the modified Shields curve $\tau^*_{c\ mod}$. For step-pool streams with a gradient of 0.09, τ^*_{cbf} reached values of 0.57 to 0.67 which are about 10 and 20 times higher than $\tau^*_{c\ orig}$ and $\tau^*_{c\ mod}$, respectively (Table 2). For the most mobile study streams (mixed plane-bed and pool-riffle morphology), measured values of τ^*_{cbf} are similar to τ^*_c values measured or predicted for data compiled by Bathurst et al. (1983), Shvidchenko and Pender (2000), Lamb et al. (2008), and Recking (2009); however, for step-pool study streams, τ^*_{cbf} values are about 7 – 9 times higher.

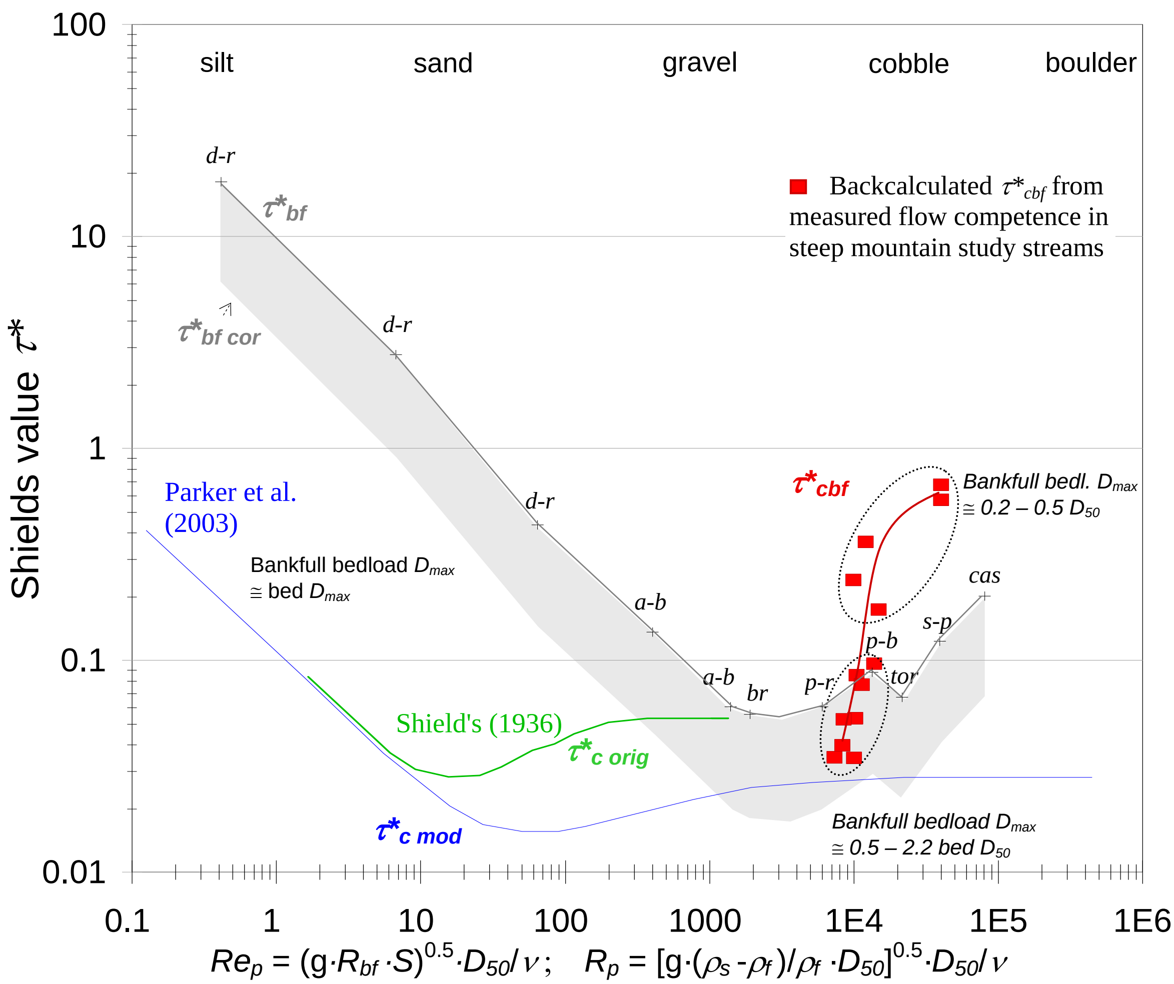

Figure 5 Four Shields-type curves plotted together: 1) The original curve $\tau^*_{c\ orig}$ (green line); 2) the modified curve $\tau^*_{c\ mod}$ (blue line); 3) Shield curve for bankfull flow τ^*_{bf} (gray band; the upper side indicates τ^*_{bf} values computed from R_{bf}, the bottom side $\tau^*_{bf\ cor}$ values computed from $R_{bf\ cor}$); 4) Critical bankfull Shields values τ^*_{cbf} computed from bedload transport measurements at 13 mountain stream study sites (red squares). Stream types are indicated by acronyms next to the cross-symbols (see bold letters in Table 1).

Comparison of τ^*_{cbf} with τ^*_{bf} and τ^*_c For the most mobile of the study streams, the computed values of τ^*_{cbf} are similar to both τ^*_c and τ^*_{bf}. In these streams, the largest bedload D_{max} particle sizes that were transported at bankfull flow ($D_{max\ bf}$) were within 0.4 to 2.1 times the bed surface D_{50} size (Table 2). This result shows that $\tau^*_{c\ orig}$, $\tau^*_{c\ mod}$, and τ^*_{bf} are similar and applicable in streams that are competent to transport particles close the bed surface D_{50} size. In the steeper plane-bed streams with occasional forced pool-riffle sequences as well as in step-pool streams and cascades, τ^*_{cbf} is considerably larger than τ^*_{bf} and particularly larger than $\tau^*_{c\ mod}$. Here, use of τ^*_{bf} and particularly $\tau^*_{c\ mod}$ can cause erroneous predictions of the bankfull mobile particle size D_{cbf} as is discussed in the next section.

Effects of using τ^*_c and τ^*_{bf} to compute the bankfull mobile particle size D_{cbf} The bankfull mobile particle size D_{cbf} computed from Eq. 3b is affected by the specific Shields value used to compute it. If τ^*_{bf} is used, D_{cbf} is equal to D_{50}, since τ^*_{bf} is based on D_{50} (thus $D_{50}/D_{cbf} = 1$, see horizontal line in Figure 6). However, this result is accurate only in those streams that transport their bed material D_{50} size at bankfull flow, i.e., in mobile gravel-bed streams. The result is not accurate in supply-limited steep mountain streams that transport particles much smaller than the bed

D_{50} at bankfull flow (see $D_{50}/D_{max\ bf}$ ratios of 0.9 – 4.9 in Table 2) and not in silt- and sand-bed streams where bankfull flow transports particles much larger than the bed D_{50} size. In mountain plane-bed and step-pool streams, the bankfull mobile particle sizes D_{cbf} predicted from τ^*_{bf} are considerably larger than the measured bankfull mobile particle sizes $D_{max\ bf}$.

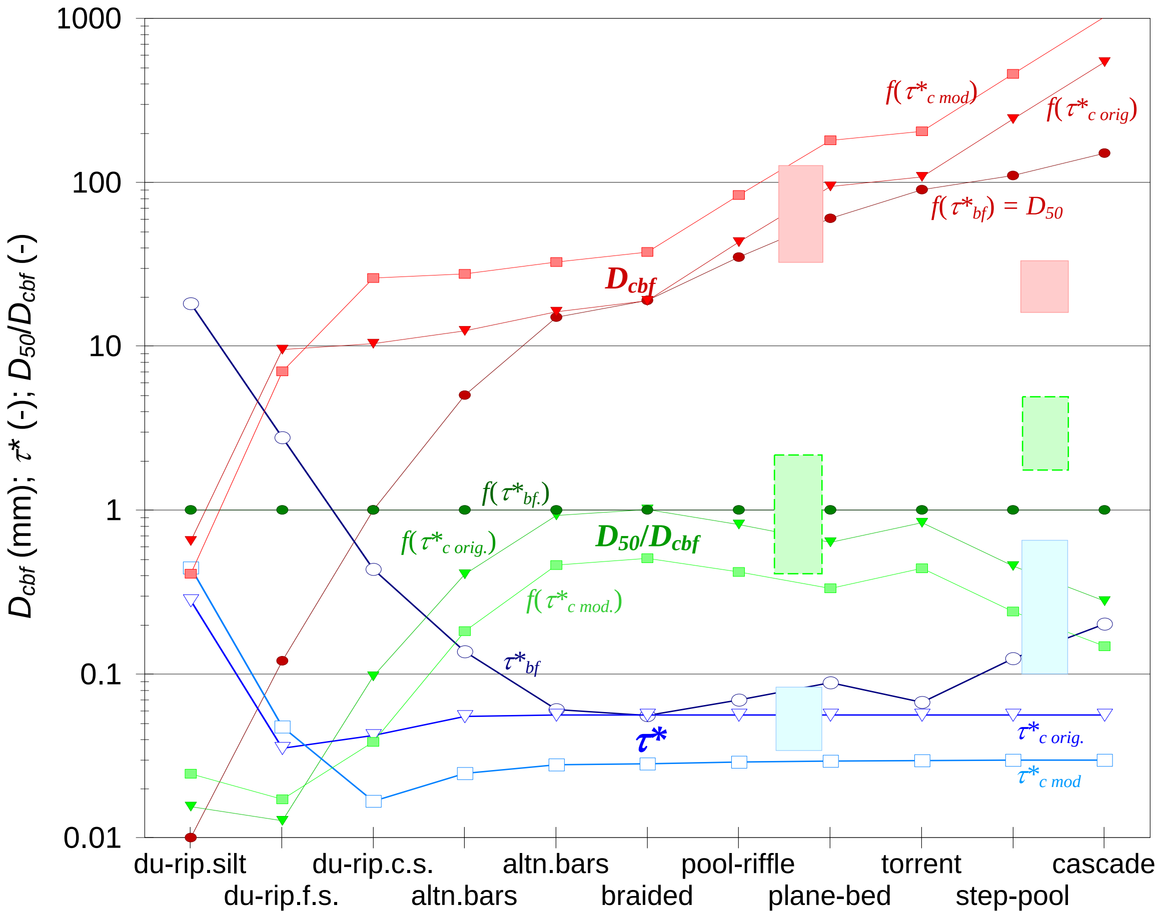

Figure 6 Comparison over the 11 model streams: 1) Three Shields-type curves τ^* (open symbols, bluish line colors); 2) Three computed bankfull mobile particle sizes (D_{cbf} , closed symbols, reddish colors); and 3) Three ratios D_{50}/D_{cbf} (closed symbols, greenish colors). Triangle symbols are used for $\tau^*_{c\ orig}$ and computations derived from it. Square symbols are used for $\tau^*_{c\ mod}$ and derived computation. Round symbols are used for τ^*_{bf} and its computa-tions. The boxes indicate the range of values obtained for τ^*_{cbf}, $D_{max,bf}$, and the ratio $D_{50/Dmax,bf}$ for the 13 study sites. Results averaged over the measured mountain plane-bed and step-pool streams (boxes) are shown for comparison.

If the original $\tau^*_{c\ orig}$ value (Shields 1936) is used to compute the bankfull mobile particle size D_{cbf}, the predicted values are similar to D_{50} only in streams where bankfull flow entrains the bed D_{50} size, i.e., in mobile gravel-bed streams. In supply-limited coarse-bedded plane-bed and step-pool streams, bankfull flows transport particles smaller than D_{50}, thus D_{cbf} is larger than D_{50}; in silt- and sand-bed streams in which bankfull flow transport particle sizes larger than the bed D_{50}, the predicted D_{cbf} size exceeds D_{50} by 1 to 2 orders of magnitude (Figure 6). The ratio of D_{50}/D_{cbf} is near unity for mobile gravel-bed streams, but drops below 1 for supply-limited plane-bed and step-pool streams as well as cascades. For silt- and sand-bed low gradient streams, the ratio D_{50}/D_{cbf} drops to values as low as 0.01. If modified Shields values $\tau^*_{c\ mod}$ are used to compute D_{cbf}, the predicted values are about twice as high as those computed from $\tau^*_{c\ mod}$, thus the ratio D_{50}/D_{cbf} is lower and near 1 only in very mobile streams in which the bed D_{50} is transported. Values of D_{cbf} discussed above are somewhat lower if computed from $\tau^*_{bf\ cor}$; based on the estimate for roughness correction used in this study ($R_{bf\ cor} = {}^1/_3\ R_{bf}$), the D_{cbf} would be ${}^1/_3$ of the reported value. Similarly, if based on Shields values $\tau^*_{c\ mod}$, computed values of D_{cbf} are twice as high as those computed from τ^*_{c}

$_{orig}$, while the ratio D_{50}/D_{cbf} is only half as large. These computations show that when the ratio D_{50}/D_{cbf} is computed over a wide range of stream types (Olsen et al. 1997; Kaufmann et al. 2008), much of the observed inter-stream variability is caused by using unsuited Shields value τ^* for the computation of D_{cbf}. Thus, if ratios of D_{50}/D_{cbf} are used to evaluate streambed disturbance, care must be taken to select adequate Shields values for each stream type.

Different "arms" of the critical bankfull Shields curve τ^*_{cbf} This study showed a steep increase of τ^*_{cbf} with Re_p for coarse and steep study streams. The high values of τ^*_{cbf} are due to low sediment supply and high structural stability of the stream beds; coarse gravels, cobbles, and boulders are firmly stuck and wedged in the bed such that only small and medium gravels are transported at bankfull flow. However, there are several relationships between τ^*_{cbf} and Re_p. For steep streams with high sediment supply such as torrents, τ^*_{cbf} likely increases much less with Re_p (Figure 7) because a high sediment supply and the resulting poor structural stability of the bed makes even large particle sizes mobile at bankfull flow. If the bed D_{50} size is mobile at bankfull flow, the τ^*_{cbf} curve approaches τ^*_{bf}.

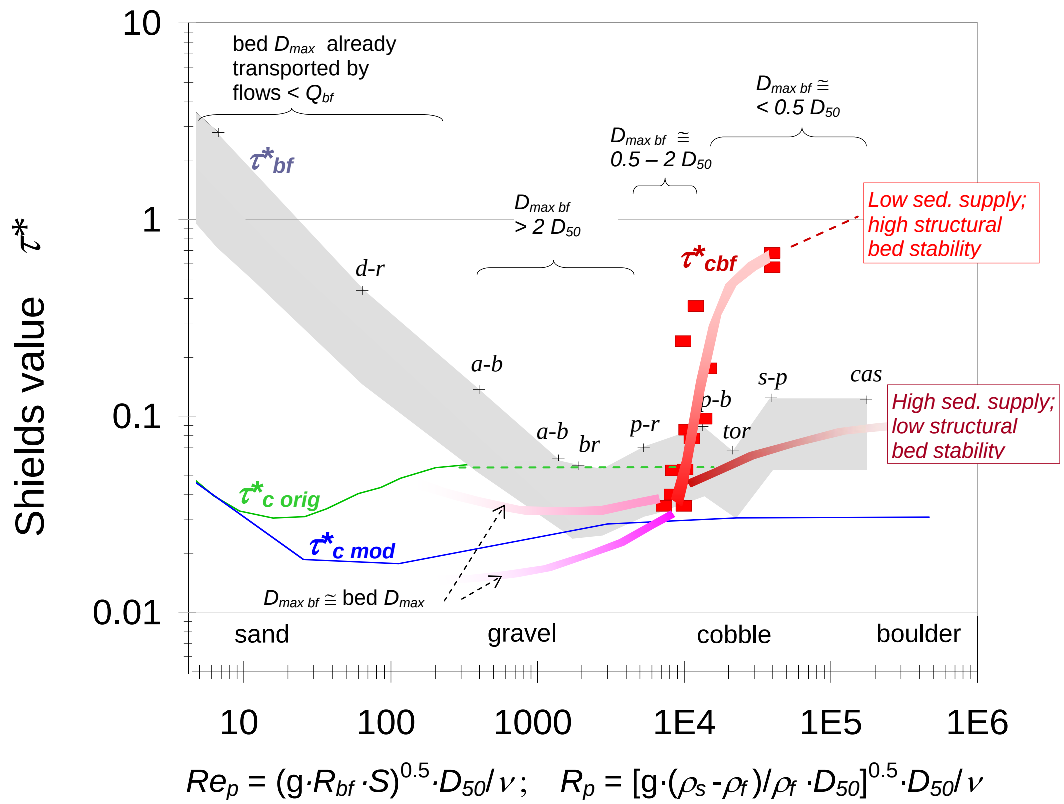

Figure 7 Annotated detail of Figure 5 showing the measured portion of the critical bankfull Shields curve τ^*_{cbf}. Also sketched (in reddish colors) are τ^*_{cbf} curves suggested for steep streams with high sediment supply as well as for streams with fine and moderate gravel beds.

One can make educated guesses about the course of the τ^*_{cbf} curve for streams finer and less steep than the study streams. For streams with medium and fine gravel beds one may assume that the largest bankfull mobile particle size $D_{max\ bf}$ exceeds the bed D_{50} size, and that consequently the τ^*_{cbf} curve will drop below the bankfull τ^*_{bf} curve. The higher the mobility of the bed (for example due to a large amount of sand in a gravel bed), the further τ^*_{cbf} will drop below τ^*_{bf}. Conversely, if bed mobility is reduced (e.g., due algae cover or a mud cake on the bed surface), τ^*_{cbf} may remain above τ^*_{bf}. The τ^*_{cbf} curve reaches its lower end (i.e., its lowest Re_p values) when the streambed particles are so fine that the largest bankfull mobile particle size $D_{max\ bf}$ approaches the bed D_{max} particle size. For sand- and fine gravel-beds, flows less than bankfull have the competence to transport the largest bed particles, and the concept of bankfull mobility no longer applies. The upper end of the τ^*_{cbf} curve is reached (i.e., highest Re_p) when a channel ceases to experience a flow regime that regularly includes bankfull flows, when a bankfull channel does not exist, and when gravity alone moves particles downstream.

The discussion above shows that bankfull critical Shields values τ^*_{cbf} are in no way fixed values but differ among streams based on sediment supply and the degree of "stuckness" or mobility of bed particles. A bankfull critical Shields curve τ^*_{cbf} has a lower and upper end and applies only to streams that have bankfull channels, experience bankfull flow now and then and where the force of water, not gravity alone, is required to move particles. Since the bankfull critical Shields curve τ^*_{cbf} indicates the largest bankfull mobile particle size, it is implied that particles larger than those mobilized as present in the bed. In silt- and sand-bed streams, the bed D_{max} particle size is entrained in flows much lower than bankfull, and bankfull critical Shields values τ^*_{cbf} do not apply here.

SUMMARY AND CONCLUSION

- Shields-type curves ($\tau^*_{c\ orig}$, $\tau^*_{c\ mod}$, τ^*_{bf}, and τ^*_{cbf}) differ among stream types. Differences are largest among:
  - Curves for τ^*_{c} and τ^*_{bf} in silt- and sand-bed streams (τ^*_{bf} of 1 and 10 for sand- and silt-bed streams were also suggested by Dale and Friend (1998), Garcia (2000), Church (2002), and Parker et al. (2003).
  - Curves for τ^*_{c}, τ^*_{bf}, and τ^*_{cbf} in cobble-bed step-pool streams
- Contrary to Shields values $\tau^*_{c\ orig}$ and $\tau^*_{c\ mod}$, critical bankfull Shields values τ^*_{cbf} are not constant at high Reynolds numbers, but increase steeply with stream gradient S and coarseness, as suggested in other studies (e.g., Rosgen 1996; Shvidchenco and Pender 2000; Mueller et al. 2006; Lamb et al. 2008; Recking 2009).
  - τ^*_{cbf} for coarse gravel beds in mixed plane-bed and pool-riffle streams covered the range 0.03 - 0.085,
  - τ^*_{cbf} for coarse gravel plane-bed streams with forced pool-riffle sequences covered the range 0.05 - 0.36,
  - τ^*_{cbf} for coarse gravel and cobble step-pool covered the range 0.17 – 0.67
- Curves for τ^*_{c}, τ^*_{bf}, and τ^*_{cbf} are least diverse for mobile gravel-bed streams where τ^*_{c}, τ^*_{bf}, and τ^*_{cbf} nevertheless extend over a 3-fold range (0.03 to 0.1).
- Very high values of τ^*_{cbf} (0.2 – 0.7) are required to compute the bankfull mobile particle size $D_{max,bf}$ in step-pool streams. Use of Shields value τ^*_{c} (0.03 – 0.06) overpredicts $D_{max,bf}$ by about an order of magnitude in these stream types.
- Bankfull mobile particle sizes can be predicted from the original Shields curve τ^*_{c} or its variants only in gravel-bed streams that transport their bed surface D_{50} size at bankfull flow, but not in supply-limited mountain streams.
- The concept of "bankfull entrainable particle size" does not apply to silt- and sand-bed where the largest bed particles are entrainable at flows much smaller than bankfull.

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