AN EXPERIMENTAL STUDY OF SAND TRANSPORT OVER AN IMMOBILE GRAVEL SUBSTRATE

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Abstract The effects of a stepwise addition of sand to an immobile gravel bed on the sand transport rate and configuration of the sand bed were investigated in a laboratory flume channel. Detailed measurements of sand transport rate, bed texture, and bed topography were collected for four different discharges (Fr~0.2, 0.3, 0.5, and 0.6) for each sand and gravel bed mixture. Sand transport was measured using both physical samples and a density cell. The elevation of the sand relative to the gravel and percent exposure of the sand bed were both evaluated in predicting the sand transport rate. For the highest two discharges the sand amalgamated into a small number of large, slow moving bed forms. A mechanism for this phenomenon is proposed. A collapse of the transport data was accomplished by relating the sand transport rate to a power function of the bed shear stress scaled by the mean size of the bed sediment, with the critical shear stress adjusted by the normalized height of the sand relative to the gravel bed and the flow rate.

INTRODUCTION

Sediment on the bed of rivers downstream of dams often becomes armored with gravel and deficient in sand sized sediment. Sand tends to be intermittently introduced to these armored streams and its transport in a generally unmoving coarse substrate is difficult to predict. Rather than moving as conventional bed load or suspended load, this sand is often transported between the immobile gravel grains as interstitial load. This type of situation exists on the Colorado River in the Grand Canyon (Topping et al., 2000) and in other streams downstream of dams. Reservoir flushing, dam by-passing, or dam removal may provide a large amount of sand to armored gravel bed channels.

Many streams have a bed material composed of a bimodal mixture of sand and gravel. For low bed shear stresses the sand portion of the bed material is entrained before the gravel begins to move. There may be a substantial range of flow strengths over which only the sand and fine gravel portions of the bed material are entrained and transported. The sand fraction of the bed material on the main channel of Goodwin Creek is an example of a stream in which the sand fraction is entrained and transported before the gravel fractions of the bed material (Kuhnle, 1992, 1993). Prediction of sand transport in bed materials with immobile gravel substrates is complicated by the variable percent exposure and depth of the sand bed and the interactions of the flow and configuration of the sand and gravel in the variable roughness bed. For some combinations of flow and sediment sizes, the sand in the interstices of the gravel sediment may be shielded from the shear stress of the flowing water, while for other cases the local
perturbations of the flow caused by the immobile gravel may act to increase the shear stress on
the interstitial sand. These competing effects must be resolved before a reasonable prediction of
sand transport may be made under these conditions.

There have been a number of studies that have investigated the effect of sand on the transport of
sand-gravel sediment mixtures (Iseya and Ikeda, 1987; Ferguson et al., 1989; Kleinhans, 2002;
Curran and Wilcock, 2005). This study is different in that the sand and gravel sediments are
separated by a large difference in size and the gravel fraction is immobile. A similar study by
Grams and Wilcock (2007) considered the suspended transport of fine sand over immobile
gravel-sized hemispheres. In this study, however, the conditions of transport of the sand fraction
were broadened to include cases in which the sand was transported exclusively as bed load and
cases in which the sand was transported as a combination of suspended and bed load through
natural gravel sediment.

Part of this study consisted of the evaluation of velocity profiles and shear stresses as affected by
variable sand elevations in a gravel bed (Wren et al., 2009). The focus here, however, will be on
the transport of sand over a stationary gravel bed in a laboratory flume. Sand transport rate was
measured in a series of experiments in which the elevation of the sand relative to the gravel in
the flume system was increased. The bed forms that occurred as the sand bed elevation
increased were studied and a method to predict the sand transport rate in a channel with an
unmoving gravel fraction was developed. A method to predict the concentration of suspended
sand in this system was also investigated.

EXPERIMENTAL METHODS

All experiments were conducted at the National Sedimentation Laboratory in an adjustable slope
15 m long, 0.36 m wide, 0.45 m deep flume channel in which sediment and water were
recirculated. The sand used in the experiments ranged in size from 0.1 to 0.5 mm in diameter
with a median grain diameter of 0.30 mm, and \((D_{84}/D_{16})^{1/2} = 1.35\). The gravel substrate, which
was not transported in the experiments, consisted of a randomly placed 0.20 m thick layer
beginning 1.2 m downstream of the head box of the channel for 13.4 m to just upstream of the
tail box. The gravel ranged in diameter from 27 to 52 mm with a median diameter of 35.0 mm
and \((D_{84}/D_{16})^{1/2} = 1.15\). The upstream 1.2 m of the flume channel consisted of a false bottom
with a single layer of gravel glued down so as to prevent its entrainment by the high shear stress
of the developing boundary layer of the flow.

Flow depth ranged from 0.20 to 0.23 m in the experiments and was measured as the difference
between the height of the bed and the water surface as determined from stream parallel transects
measured in the center of the channel over the downstream 8 m using in-air and underwater
acoustic distance measuring instruments mounted on an instrument carriage which rode on rails
above the flume channel. Water surface slope was determined as the sum of the slope of the
water surface transect relative to the flume rails and the slope of the flume channel. Discharge in
the channel was measured using a calibrated Venturi meter read by a pressure transducer in the
0.15 m diameter sediment and water return pipe. Four discharges, nominally 20, 30, 50, and 65
l/s were imposed on each of the 11 sand an gravel beds. Shear stress was calculated for each
experimental run as
\[ \tau = \rho ghS \]  

where \( \rho \) is the density of the water, \( g \) is the acceleration of gravity, \( h \) is the mean depth of the flow, and \( S \) is the slope of the water surface. The boundary shear stress was corrected for the effect of the sidewall of the flume using the relation of Chiew and Parker (1994).

Sand was added to the flume channel near the downstream end in doses of 20 or 40 kg over a few hours using a vibrating sediment feeder. Initially the sand was removed from transport rapidly as the pores in the gravel bed were filled from the bottom up. The sand front gradually moved downstream until the entire bed was filled to a similar level. Transport of the sand began when 300 kg had been added to the channel. The elevation of the sand bed after the addition of 300 kg of sand was measured with a point gauge at 51 points over an 8 m length of the channel. The mean elevation of the sand bed was found to be 5.0 cm below the maximum gravel elevation. After successive sand additions the sand levels above the -5 cm level were calculated using the porosity of the gravel and sand and the dimensions of the gravel bed in the flume. The relation between the porosity of the gravel and the elevation below the maximum gravel level was measured in an acrylic cylinder filled with the same mixture of gravel as in the flume. The increase in water depth resulting from incremental additions of known volumes of water to the cylinder allowed the changing porosity of the gravel layer to be determined accurately. This empirically determined porosity relation was used to calculate the mean sand elevation in the channel after each sand addition above the -5.0 cm level. Porosity was found to be constant below -3.0 cm or approximately 1 \( D_{50} \) below the maximum gravel elevation. Flow and sediment transport measurements were collected from 11 different sand bed elevations ranging from 5.0 cm below the mean maximum elevation of the gravel to 0.13 cm above.

Sediment transport was measured using a 12.7 mm (½ inch) inside diameter sampling nozzle mounted in the center of the return pipe just downstream of the pump impellor. The flow velocity into the sampling nozzle was adjusted to match the mean velocity of the flow in the return pipe (isokinetic) to avoid biasing the sample. The discharge in the 1.3 cm diameter sampling line was measured by collecting the flow in a 20 l vessel over a measured period of time and determining the mass of the water on an electronic balance. Total sediment transport rate was determined using two methods depending on the magnitude of the transport rate in the experiment. The first method, used for low sediment transport rates, consisted of catching all sand passing through the sampling nozzle using a 0.053 mm sieve in the stream of water from the sampling line. For higher sediment transport rates, sediment transport in the sampling line was measured by recording the output from a density cell (Dynatrol model CL-10 HYS) at 1 Hz. The density cell consisted of a vibrating u-shaped tube and was calibrated for sediment concentration and flow rate using the same sand used in the experiments. The range of the calibration conditions included the flows and concentrations observed in this study. The mean sediment concentration from the physical samples or from the density cell, which were collected for at least 4 hours, were multiplied by the mean flow rate from the Venturi meter to obtain the total sand transport rate (mass/time).

Suspended sediment concentrations were sampled using 3.2 mm nozzles (inside diameter) at four elevations above the bed in one vertical line in the center of the channel simultaneously. The
samples were collected over the range of $z/h$ from 0.1 to 0.67, where $z$ is the distance above the local bed, and $h$ is the depth of flow. The samples were collected 3 m upstream from the tail box of the channel. Four sets of four samples were collected isokinetically for two minutes with approximately two minutes between sample sets. Suspended sediment samples were collected only during experiments with flow of 50 and 65 l/s due to the dearth of suspended sediment in the experiments with lower flows.

**EXPERIMENTAL RESULTS**

**Sediment Transport Relation for Sand** Total sand transport rate and bed shear stress for the eleven sand bed elevations and four flow discharges were found to be poorly correlated, as expected (Fig. 1). To generalize the results of the transport rate for the 11 different beds, the mean grain size of each sediment bed was calculated based on the fraction of sand exposure. Sand transport rate was non-dimensionalized using:

$$q^*_s = \frac{q_s}{\left[(s-1)gD_{mb}\right]^{\frac{1}{2}} \rho_s D_{mb}}$$

(2)

where $q_s$ is the sediment transport rate in mass per time per unit width, $s = \rho_s / \rho$, $\rho_s$ is the density of the sediment grains, $\rho$ is the density of the water, $g$ is the acceleration of gravity, and $D_{mb}$ is the weighted mean diameter of the bed material sediment, which was calculated assuming the fraction of the pore space in the gravel at that elevation was filled with sand as determined from the empirically-derived porosity relation discussed above. The weighted mean bed material size ($D_{mb}$) was calculated using the median diameter of the sand and gravel. The bed shear stress ($\tau_b$) was scaled by the critical shear stress of the mean bed material sediment size ($\tau_{bc}$) using the relation of Miller et al. (1977). The relation between dimensionless sediment transport
rate and relative shear stress shows a high degree of correlation (Fig. 2), however, there is still an appreciable spread in the data (~100 x), particularly for lower relative shear stresses.

A factor was defined to correct for the differences in the elevations of the sand and the gravel. The relation between dimensionless transport rate and relative shear stress from the experiments with sand elevation 0.13 cm above the top of the gravel was used as the reference relation for the other beds:

$$ q_s^* = 0.00087 \left( \frac{\tau_h}{\tau_{bc}} \right)^{2.50} $$  \hspace{1cm} (3)

Next a depth correction factor ($c_d$) was defined to adjust the value of the critical shear stress for the relations between dimensionless transport rate and relative shear stress:

$$ q_s^* = 0.00087 \left( \frac{\tau_h}{\tau_{bc} / c_d} \right)^{2.50} $$  \hspace{1cm} (4)

A normalized sand bed elevation ($Z_s$) was defined with the zero level at 50 mm below the maximum gravel elevation:

$$ Z_s = \frac{50.0 + E_s}{50.0} $$  \hspace{1cm} (5)

where $E_s$ is the distance of the sand bed relative to the maximum elevation of the gravel bed. $E_s$ has a negative sign when the sand is below the top of the gravel and positive when it is above the top of the gravel. The sediment diameter of 50.0 mm represents the size of the gravel that 99 percent is finer than by weight. When the sand bed was 50 mm below the top of the gravel only a very small amount of sand was transported at the highest flow, and thus the depth correction factor ($c_d$) approached zero and the effective critical shear stress became large. Below this level the sand was essentially removed from the transport zone.

Equation (4) was solved for $c_d$ for each of the pairs of shear stress and transport rate measured for the 11 sand bed elevations and related to $Z_s$. In addition to being related to $Z_s$, the depth correction factor was also found to be a function of flow rate (Fig. 3). With the assumption that the depth correction factor ($c_d$) tends to unity, a sigmoid relation of $Z_s$ for each flow rate gave a reasonable prediction of the calculated values of $c_d$:

$$ c_d = \frac{1}{1 + \exp \left( \frac{- (Z_s - b) / 0.15}{0.15} \right)} $$  \hspace{1cm} (6)
where $b$ is a fitted constant which may be predicted by the mean flow velocity at 10 percent of the flow depth (Fig. 4). The incorporation of the depth correction factor ($c_d$) markedly improves the correspondence of the equation (4) and the measured transport data (Fig. 5).

![Graph of $c_d$ vs $Z_s$](image1)

**Figure 3** Depth correction factor versus $Z_s$ for three different flows. The same relation to predict $c_d$ was used for the 20 and 30 l/s flows.

![Graph of sigmoid function parameter $b$](image2)

**Figure 4** Prediction of the “$b$” parameter in the sigmoid equation (6).

![Graph of sand transport relation](image3)

**Figure 5** Sand transport relation with sand depth correction factor.

**Bed Forms** Periodic fluctuations of the total sediment load were observed in the data collected by the density cell for the seventh sediment bed ($E_s = -1.6$ cm). As the elevation of the sediment bed was increased, periodic fluctuations of the total sediment load were also recognized from sediment concentrations measured with the density cell for the lower flows. During the eighth bed ($E_s = -1.2$ cm), the fluctuations measured by the density cell and bed height records became very regular in amplitude and spacing (Figs. 6, 7). It was observed that a single long and low bed form was present over the length of the flume channel. In later experiments, the bed forms had a shorter spacing and multiple forms were present on the flume channel at one time resulting...
in multiple successive peaks in concentration interspersed with periods of lower concentration (Fig. 8).

Figure 6 Fluctuations in sediment concentration as collected by the density cell on the eighth bed with flow discharge of 65 l/s and $E_s = -1.20$ cm.

Figure 7 Sediment transport fluctuations measured by density cell on the eight bed with flow discharge of 50 l/s. Data have been filtered using a 15-term moving average, $E_s = -1.20$ cm.

Figure 8 Sediment transport fluctuations measured by density cell on the tenth bed with flow discharge of 50 l/s. Data have been filtered using a 15-term moving average, $E_s = -0.40$ cm.
A conceptual model was developed for the migration of a single bed form over the non-moving gravel. This model begins in the trough of the bed form where the level of the sand is significantly below the top of the gravel ($Z_s \to 0$), progresses up the stoss of the bed form ($0 < Z_s < 1$), and finally to the crest of the form ($Z_s > 1$) (Fig. 9).

1. $Z_s \to 0$, There is nearly instantaneous infiltration of sand traveling near the bed into the gravel with sand going to the lowest level not filled with fine material. Smaller grains and grains injected high enough into the flow by the crest pass by this region. No net transport of sand through the interstices occurs past this point. All sand transport is as suspended load. Grain shear is at a minimum, turbulence is high, suspended load and total shear are at peak levels.

2. $0 < Z_s < 1$, The concentration of suspended sediment begins to decrease as the sand surface rises above the $D_{99}$ level. Bed load transport is small as particles in transport near the bed become trapped by the unfilled gravel bed. Grain shear is increasing, while turbulence and total shear are decreasing.

3. $Z_s > 1$, The bed load and total load peak as the gravel is buried and the bed becomes smoother. The concentration of suspended load, turbulence and total shear are at minimum values. The accelerated flow velocity over the crest injects sediment into the flow downstream of the crest and causes downstream scour of the sand in the gravel. That part of the bed form above the level of the gravel is caused by the disparity in transport rate between gravel and sand dominated areas. Upstream areas ($0 < Z_s < 1$) have higher rates of suspended sediment transport so sand catches up to the area where the gravel is nearly filled with sand ($Z_s \to 1$).

Figure 9 Schematic of sand bed form in gravel. The yellow dotted region represents the sand filled portion of the bed and the brown circles represent the gravel. The three zones are described in the text.
**Suspended Sediment Transport**  Measured suspended sediment concentrations increased as the normalized sand bed elevation increased (Fig. 10). Concentration distributions corresponded reasonably well with the distributions generated using eq 8 (see below) with the suspended sample collected nearest to the bed used for the reference concentration. The correction factor developed by Grams and Wilcock (2007) was used along with the entrainment function of Garcia and Parker (1991). The Garcia and Parker function calculates a near bed sediment concentration for sand bed streams and the correction factor of Grams and Wilcock (2007) adjusts the predicted value for sand in immobile gravel beds:

\[
\bar{C}_a = \varepsilon \hat{E}_g \tag{7a}
\]

\[
\varepsilon = \left(1 + e^{-18(z_s - 0.2)}} \right)^{-1.1} \tag{7b}
\]

\[
\hat{z}_s = \frac{z_s}{r_b} \tag{7c}
\]

where \(\bar{C}_a\) is the sediment concentration (volumetric) at 0.05 \(y/h\) for an immobile gravel bed partially filled with sand, \(\varepsilon\) is the correction factor, \(\hat{E}_g\) is the reference concentration at 0.05 \(y/h\) predicted for a sand bed by the Garcia and Parker relation, \(\hat{z}_s\) is the normalized average sand bed elevation, \(\bar{z}_s\) is the spatially averaged sand bed elevation, and \(r_b\) is the height of the immobile coarse grains. For this study the height of the immobile coarse grains was assumed to be \(\frac{1}{2}\) the height of the \(D_{99}\) size of the gravel (25 mm) following the recommendation from Grams and Wilcock (2007). The corrected values from the Garcia and Parker (1991) relation (eq 7a) were used as the reference value in a modified form of the Rouse equation (Van Rijn, 1984) to predict a vertical distribution of suspended sediment concentration for the 50 and 65 l/s experiments in this study:

\[
\frac{C_i}{C_{ai}} = \left( \frac{a}{h-a} \right)^{\frac{y}{y/(h-a)}} \quad \text{for} \quad \frac{y}{h} < 0.5 \tag{8a}
\]

\[
\frac{C_i}{C_{ai}} = \left( \frac{a}{h-a} \right)^{\frac{y}{y/(h-a)}} \exp^{-4Z(y/h-0.5)}, \quad \text{for} \quad \frac{y}{h} \geq 0.5 \tag{8b}
\]

where \(C_i\) and \(C_{ai}\) are the sediment concentration for the \(i^{th}\) grain size group at the distance \(y\) from the bed and at the reference elevation \(a\), (Kuhnle and Wren, 2009) respectively, and

\[
z = \frac{\omega_i}{\beta \kappa u_*} \tag{9a}
\]

where

\[
\omega_i = \frac{\lambda_i}{D_i}
\]

\[
\beta = \frac{1}{\kappa u_*}
\]

\[
\kappa = \sqrt{\frac{2}{3}}
\]

\[
u = \frac{1}{3}
\]

\[
\lambda_i = \frac{1}{\sqrt{2}}
\]

\[
D_i = \frac{1}{2}
\]
\[ \beta = 1 + 2 \left[ \frac{\omega_i}{u_*} \right]^2 \text{ for } 0.1 < \frac{\omega_i}{u_*} < 1.0 \] (9b)

(Van Rijn, 1984), with \( \omega_i \) the fall velocity of grains of the \( i^{th} \) size group, \( u_* = \sqrt{\frac{\tau_0}{\rho}} \), \( \tau_0 \) is the boundary shear stress calculated from the depth slope product, \( \kappa \) is the von Kármán constant and assumed equal to 0.4, and \( \rho \) is the density of the water. The predicted suspended sediment concentrations generally do not correspond well with the measured values (Fig. 10). The predicted concentrations for low values of \( Z_s \) were less than measured values, while for larger values of \( Z_s \) the predicted values were greater than measured values.

![Figure 10. Comparison of measured sediment concentrations and concentrations generated from predicted reference concentrations (Grams and Wilcock, 2007) coupled with eq (8). The blue line used a measured reference concentration from the -1.60 cm sand bed. The red lines used reference concentrations calculated using the technique of Grams and Wilcock (GW) and equation (8). The value in the caption to the right of GW, is the value of \( \varepsilon \) from eq 7.](image)

**CONCLUSIONS**

Transport of sand in the interstices of the immobile gravel bed began when the elevation of the sand bed was about 1 maximum gravel diameter \( (D_{99} = 50 \text{ mm}) \) below the maximum elevation of the gravel. Sand transport increased rapidly as the elevation of the sand bed increased.
Total sand transport rate was found to be a function of the weighed mean grain size of the bed surface (calculated using percent sand exposed), the normalized sand bed elevation relative to the gravel bed, and the flow velocity at $z/h = 0.10$.

Bed forms with a nearly constant spacing and migration rate occurred when the sand bed elevation was 1.6 cm or less below the maximum gravel elevation ($E_s = -1.6$ cm). A conceptual model of the migration of the bed forms was developed which invokes the modes of transport and relative rates associated with the different elevations of the sand bed relative to the gravel.

Vertical suspended sediment concentration distributions, calculated using a form of the Rouse equation (Van Rijn, 1984) with the reference concentration calculated using the near-bed concentration correction factor (Grams and Wilcock, 2007) and the entrainment relation of Garcia and Parker (1991), were found to compare poorly to the suspended sediment concentrations from samples collected for this study.

REFERENCES


