A TWO DIMENSIONAL FINITE ELEMENT HYDRODYNAMIC RIVER MORPHOLOGY AND GRAVEL TRANSPORT MODEL

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Abstract A depth-averaged finite element two-dimensional hydrodynamic-morphological and gravel transport model is presented. Based on River2D by Steffler et al. (2002) of the University of Alberta, River2D-Morphology (R2DM) uses an unstructured mesh to simulate flow hydraulics, sediment transport for uni-size and mixed size sediment, and morphological changes of a river over time. Changes in bed elevation are calculated by solving Exner's equation for sediment mass conservation. For uni-size sediment transport, R2DM uses transport functions of Meyer-Peter-Muller, Engelund-Hansen, Van Rijn or an empirical formula. For mixed size sediment transport, the function of Wilcock and Crowe (2003) is used. Secondary flow effects are modeled using the technique of Struiksma et al. (1985) with a unique algorithm that calculates the radius of curvature from the streamlines. R2DM is capable of exploring the dynamics of grain size distributions such as vertical sorting and armoring and can calculate the fraction of sand on the bed surfaces of rivers and streams. Results have been verified with experimental data for aggradation and downstream fining (Paolo, 1992; Seal, 1992; Toro-Escobar, 2000); for degradation (Ashida and Michiue, 1971); and for dispersion (Cui and Parker, 2003). R2DM can be potentially applied to actual rivers to simulate morphodynamic applications such as dam removal, dam breaks, river restoration projects, the design of hydraulic structures, and fish habitat modeling.

INTRODUCTION

Numerical models provide the basis for understanding the hydrodynamic and geomorphic conditions in river ecosystems and are a valuable tool towards solving complex issues in river and environmental engineering. Using numerical models provides the ability to simulate possible scenarios under altered hydraulic or watershed conditions, allowing scientists and engineers to provide solutions to existing problems and to anticipate future issues before they happen.

Most of the sediment transport models used in river engineering are one dimensional, especially those used for long-term simulation of a long river reach (e.g Hoey and Ferguson, 1994; Ferguson and Church, 2009). One dimensional models generally require the least amount of field data for calibration and testing. The 1D numerical solutions are more stable and require less computational time than 2D simulations, however, 1D models are generally not suitable when changes across the channel width are important such as those produced by curvature effects in bends or the presence of multiple channels (braided or anabranching rivers).

This paper will present the updated version of River2D-Morphology (R2DM), a depth averaged 2D morphodynamic model previously limited to model uni-size sediment transport (Vasquez *et al.*, 2007, 2008). The additional features to R2DM include: 1) modeling mixed sized sediments; 2) modeling of sediment sorting; 3) modeling of moving bed-forms and 4) and a new method to calculate the radius of curvature in unstructured meshes. In this present contribution, we employ four experiments to demonstrate these features.

RIVER2D

River2D (Steffler and Blackburn, 2002) is a two-dimensional, depth-averaged finite element model developed for use on natural streams and rivers and is freely available from www.River2D.ca. It solves the basic mass conservation equation and two components of momentum conservation. Outputs from the model are two velocity components (in x and y directions) and a depth at each node. Velocity distributions in the vertical are assumed to be uniform and pressure distributions are assumed to be hydrostatic. Three-dimensional effects, such as secondary flows in curved channels, are not calculated. River2D calculates the hydrodynamics based on the 2D vertically averaged St. Venant equations:

$$\frac{\partial h}{\partial t}+\frac{\partial q_x}{\partial x}+\frac{\partial q_y}{\partial y}=0 \quad (1)$$

$$\frac{\partial q_x}{\partial x}+\frac{\partial (uq_x)}{\partial x}+\frac{\partial (vq_x)}{\partial y}+\frac{g}{2}\frac{\partial h^2}{\partial x}=gh\left(S_{ox}-S_{fx}\right)+\frac{1}{\rho}\left(\frac{\partial (h\tau_{xx})}{\partial x}+\frac{\partial (h\tau_{xy})}{\partial y}\right) \quad (2)$$

$$\frac{\partial q_y}{\partial x}+\frac{\partial (uq_y)}{\partial x}+\frac{\partial (vq_y)}{\partial y}+\frac{g}{2}\frac{\partial h^2}{\partial y}=gh\left(S_{oy}-S_{fy}\right)+\frac{1}{\rho}\left(\frac{\partial (h\tau_{yx})}{\partial x}+\frac{\partial (h\tau_{yy})}{\partial y}\right) \quad (3)$$

Where h is the water depth, u and v are the vertically averaged velocities in x and y respectively, and $q_x = uh$ and q_y $=vh$ are the discharge intensities.

S_{ox}, S_{oy} are the bed slopes; S_{fx}, S_{fy} are the friction slopes; τ_{xx}, τ_{xy}, τ_{yx}, τ_{yy} are the components of turbulent stress tensor; g is the gravitational acceleration and ρ is the water density. Further details of River2D can be found in Steffler and Blackburn (2002).

River2D-Morphology: River2D Morphology (R2DM) was initially developed by Vasquez (2005) so that morphodynamic changes of a river bed could be simulated (Vasquez *et al.*, 2007, 2008). It exists as a separate module that links to the River2D hydrodynamics by solving the bed load transport continuity equation:

$$(1-\lambda)\frac{\partial z_b}{\partial t}+\frac{\partial q_{sx}}{\partial x}+\frac{\partial q_{sy}}{\partial y}=0 \quad (4)$$

where q_{sx} and q_{sy} are the components of volumetric rate of bedload transport per unit length in x and y. λ is the porosity of the bed material, t is time and z_b is the bed elevation.

The bedload transport, q_s, can be computed using either of the empirical equations for uni-size sediment: Engelund-Hansen (Engelund and Hansen, 1967), Meyer-Peter-Müller, and Van Rijn (1984). The updated version of R2DM includes the Wilcock and Crowe (2003) equation for mixed-sized gravel-sand mixtures:

$$W_i^* = \begin{Bmatrix} 0.002\phi_i^{7.5} & \phi < 1.35 \\ 14*\left(1-\frac{0.894}{\phi_i^{0.5}}\right)^{4.5} & \phi \geq 1.35 \end{Bmatrix} \quad (5)$$

where $\phi = \dfrac{\tau}{\tau_{ri}}$, τ_{ri} is the reference shear stress for fraction i, F_i = proportion of i on the bed surface, u_* = shear velocity, q_{bi} = volumetric transport rate per unit width of size i, s = ratio of sediment to water density, g = gravity, and W_i^* is the dimensionless transport rate defined by:

$$W_i^* = \frac{(s-1)gq_{bi}}{F_i u_*^3} \quad (6)$$

The total transport rate is thus:

$$q_s = k_T \frac{u_*^3 \sum_{i=1}^{N} F_i W_i^*}{(s-1)g} \quad (7)$$

Where k_T is the transport rate factor (~0.01-100).

A gravel transport algorithm is used to update the grain size distribution of the armor (surface) layer. It assumes that the active surface layer remains at a constant thickness, L_s, set by the user (set approximately equal to 2 x D_{90}). At each time step the grain size distribution for the surface layer distribution is recalculated according to the volume of sediment entering or leaving the element. Figure 1 illustrates how sediment enters and leaves a typical element.

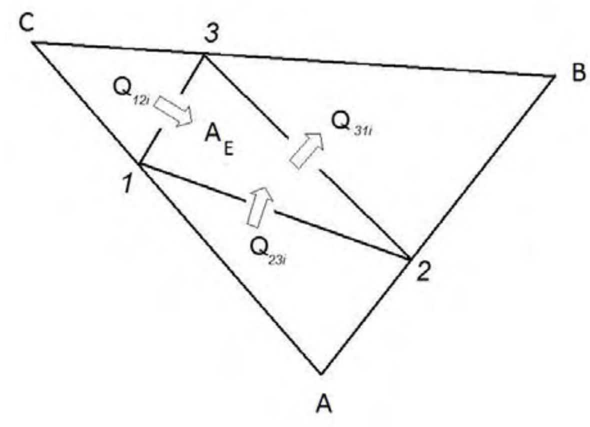

Figure 1. Diagram to illustrate sediment flux entering and leaving element with area, A_E

During aggradation, the net volume of sediment entering an element is positive and the new fraction of grain size *i* in the surface layer of the element can be determined as follows:

$$V_i = (Q_{12i} + Q_{23i} + Q_{31i})dt + (L_s - dz_b) * (1 - \lambda) F_i A_E \quad (8)$$

where V_i = volume of fraction *i* in the surface layer (L_s), F_i = surface layer fraction *i*, and L_s = surface layer thickness (~2 x D_{90}), λ = porosity, Q_{12i} = volume of sediment in fraction *i* entering or leaving through side *12* of the element per unit time.

During degradation ($\Delta z_b < 0$) sediment leaves the element and the surface layer mixes with the substrate to maintain a constant L_s. The new volume of fraction *i* is therefore:

$$V_i = (1 - \lambda) A_E \left[(L_s - dz) * F_i + dz * F_{si} \right] \quad (9)$$

where F_{si} is the fraction of grain size *i* in the substrate.

Thus, the new surface layer fraction at time step, $t+\Delta t$ is:

$$F_s^{new} = \frac{V_i}{V_{Total}} = \frac{V_i}{L_s A_E (1 - \lambda)} \quad (10)$$

This formulation accounts for the changes in surface layer composition during aggradation or degradation and satisfies mass continuity.

Simulation of Moving Bed-forms The initial numerical discretization of R2DM used a conventional Galerkin Finite Element Method, which is analogous to a centered finite difference scheme, and thus is not adequate for modeling migrating bedforms. In the updated version of R2DM, up-winding was therefore introduced into the numerical scheme to make it possible to simulate migrating bed-forms (advective transport). Up-winding is a form of numerical discretization methods for solving hyperbolic partial differential equations and utilize an adaptive or solution-sensitive finite difference stencil to numerically simulate more properly the direction of propagation of information in a flow field. A simple first order up-winding method has been incorporated into R2DM that calculates the sediment flux in each element by taking into account the flux flowing in from neighbouring elements:

$$Q = (1-UW)*Q_{downstream} + UW*Q_{upstream} \tag{11}$$

where $0 \le UW \le 1$ is the up-winding weighting factor.
UW=1 means full up-winding, and
UW=0 means no up-winding is used (conventional Galerkin Finite Element Method).

For example in Figure 1, sediment enters through sides 1-2 and 1-3, while leaving by side 2-3. Sediment fluxes through each element side will be computed as:

$$Q_{12} = (1-UW)*\left(\frac{Q_1+Q_3}{2}\right)+UW*Q_A \tag{12}$$

$$Q_{23} = (1-UW)*\left(\frac{Q_2+Q_3}{2}\right)+UW*Q_1 \tag{13}$$

$$Q_{13} = (1-UW)*\left(\frac{Q_1+Q_3}{2}\right)+UW*Q_{13} \tag{14}$$

The average bed change Δz_b at the element during a time interval Δt can therefore be calculated using:

$$\Delta z_b = \frac{\Delta t}{(1-\lambda)}*\left(\frac{Q_{12}+Q_{13}-Q_{23}}{A}\right) \tag{15}$$

where A= area of element, λ = porosity

The effect of using up-winding is illustrated in Figure 2.

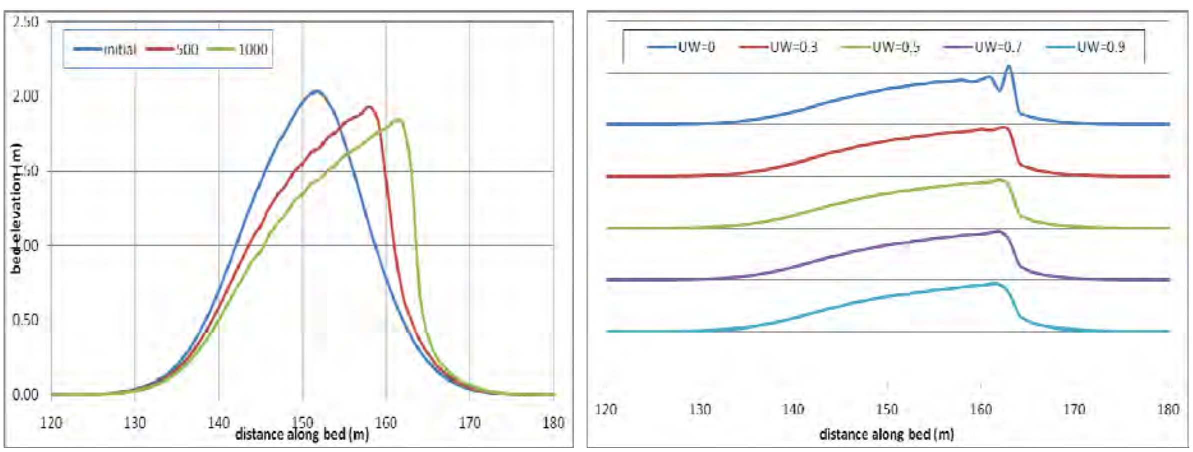

Figure 2. Simulation of migrating bed-form using up-winding and the effect of varying the up-winding coefficient on the simulation of a moving bed-form.

Bed Aggradation and Sediment Sorting The performance of the R2DM's gravel transport module was examined by simulating the experiments performed at the St. Anthony Falls Laboratory (SAFL) by Paola *et al.*, 1992; Seal *et al.*, 1997; Toro-Escobar *et al.*, 2000. Six experiments (3 narrow and 3 wide flume runs) were performed but this paper will present the results of Runs 1 and 4 only because of the limited space (the results of the other Runs are reported in Kwan, 2009). These six experiments were performed for the purpose of reproducing downstream fining of a gravel-sand mixture in response to bed aggradation. The channel used for the experiments had a depth of 1.83 m, a width of 2.74 m, length 60 m and with an initial concrete-bottom. For runs 1,2,3 and 6, the flume was narrowed to 0.305 m using a partition. The flume is ponded at its downstream reach by setting a constant water surface elevation at the downstream end, which drives channel aggradation and downstream fining. The water flow rate, Q_w, sediment feed mixture, Q_s, and the tailgate elevation ξ_d were set at constant values for each experiment; grain size of the sediment had a specific weight of 2.65.

During the course of the experiments a depositional wedge (Figure 3), having a mildly concave profile and distinct avalanche front, was formed which progressed downstream with time. Channel bed elevations were constantly monitored and the sediment was sampled at the end of each run using both the Klingeman method and a standard Wolman pebble count.

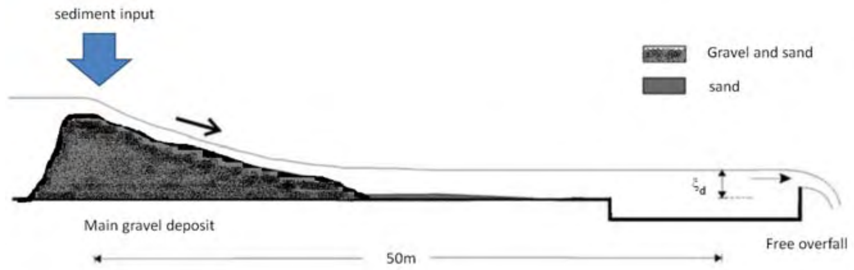

Figure 3. Experiment setup of the SAFL experiments (after Cui, 2007)

The mixed sediment transport feature of R2DM was developed to model alluvial river beds and so it was necessary to specify a grain size distribution for the bed even though the initial bed of the flume was concrete. To model the concrete bed, a surface distribution which consisted mostly of grains 16 mm and above was therefore used to simulate the concrete bed.

The narrow flume was discretized using 318 triangular elements and 320 nodes in a simple 1D layout with an average distance between nodes of 0.35m. The wide channel runs were discretized using 462 triangular elements and 282 nodes. The average distance between the nodes was set to 0.8 m. The lateral variations in the wide channel runs were introduced by altering the bed by either increasing the elevation of random nodes by 0.5 mm. It was also found that slight lateral variations in the flow arose without making these changes because of the irregularity of the mesh.

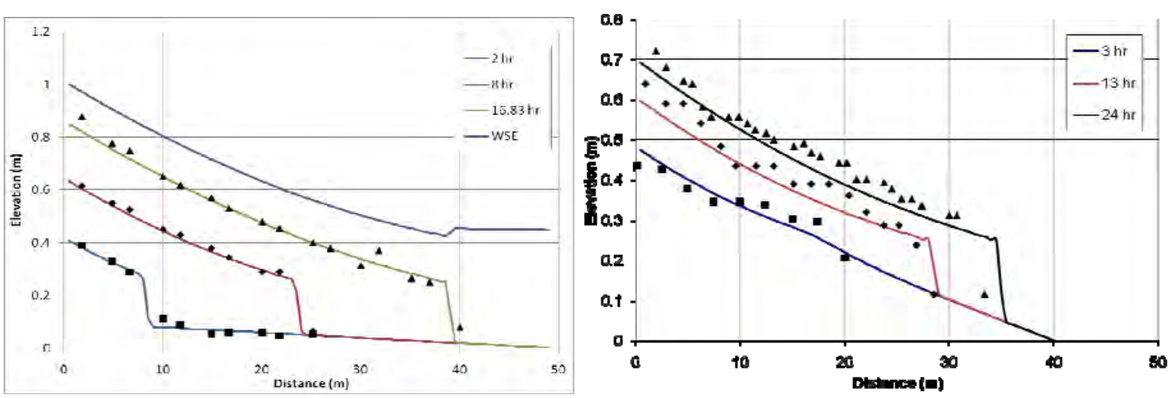

Figure 4. Simulated bed profiles shown with experimental results for Run 1 (left) and Run 4 (right) shown with final water surface elevation.

Results and Discussion of Bed Aggradation Case: The bed elevations were fine tuned to fit the experimental results by adjusting the k_T (transport rate factor) and C_{90} (where the bed roughness, $k_s = C_{90} D_{90}$) with the surface layer kept at a constant thickness of 0.05 m (this value is used by Cui, 2007). It was also found that different initial surface layer distributions (with sand content varying from 0 to 90%) had little effect on the morphological changes of the bed. The initial substrate distribution was set to the same values as the feed and had no effect on the results because no degradation occurred. For Run 1, n_k =4, and k_T=0.2 was found to give the best match to experimental results (Figure 4) while for Run 4, a good match was found with C_{90} =2.5, and k_T=0.1.

The characteristic grain sizes for Run 1 along the length of the flume are shown in Figure 5 alongside the experimental observations of Paola *et al.*(1992). It can be seen that R2DM can reproduce the observed grain size distributions as well as downstream fining.

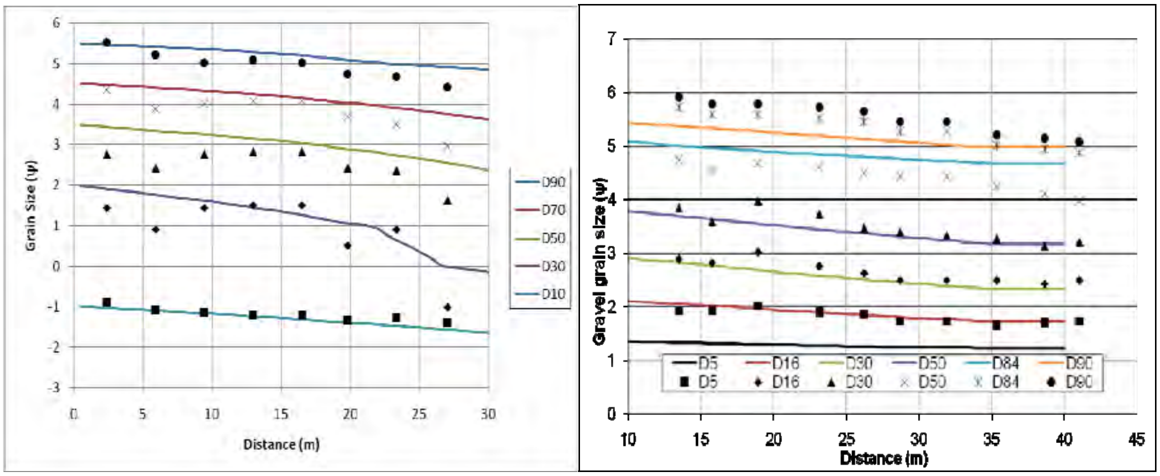

Figure 5. Simulated and observed grain size, ψ ($= \log_2 D$) for Run 1 (left) and Run 4 (right).

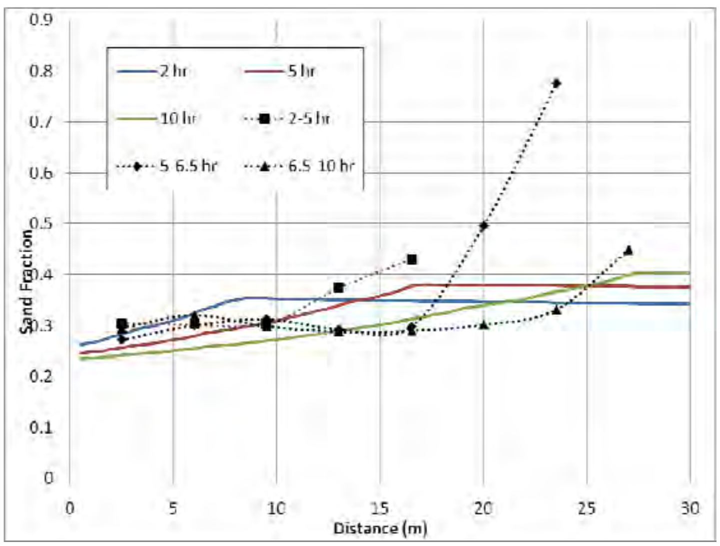

Figure 6. Simulated and observed sand fraction for Run 1. Experimental data obtained from Cui, 2007.

The simulated sand fractions (Figure 6) show that they are approximately between 5-10% of observed values in both time and space with the exception of two high sand fractions for the time interval between 6 and 14 hours in the experiment. The observed high sand fraction values are samples from a small amount of sand deposit downstream of the main depositional front (Cui, 2007).

Plan views of the final bed surface for Run 4 are shown in Figure 7 alongside with the experimental results of Toro-Escobar *et al.* (2000).

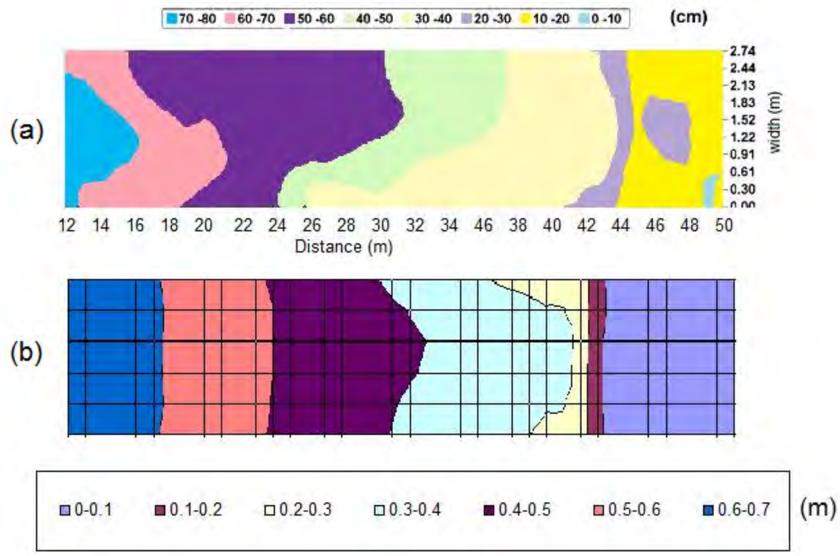

Figure 7. Perspective views of final bed surface of (a) SAFL run 4 (after Toro-Escobar *et al.*, 2000) and simulated results using R2DM.

These results demonstrate that R2DM is capable of simulating the propagation of aggradation wedges, predicting the deposition amounts, the advance rate of the wedge and downstream fining. The wide channel simulations demonstrate that R2DM can simulate two dimensional sediment transport.

Bed Degradation Bed degradation can be observed in a flume by shutting off the upstream sediment supply. Ashida and Michiue (1971) performed such experiments in a flume to investigate bed degradation and armouring downstream of a dam. The experimental flume was 20 m long, 0.8 m wide, a slope of 1% and was filled with gravel with a median size of 1.5mm. Water was fed at the inlet at 0.0314 m^3/s and the outflow was kept constant at 0.06 m. For the simulation, the flume was discretized using a one dimensional mesh of 52 triangular elements with 50 nodes. The average distance between nodes was 0.8 m.

Results and Discussion of Bed Degradation Case Good agreement with experimental results was found with C_{90} =3, k_T = 5, L_s=0.07 and UW=0. The experiments showed that the total scour process could be divided into two phases: 1) from 0-100 minutes, the bed was intensively scoured; 2) after 100 minutes, the scouring rate was reduced and an armouring layer was formed gradually. This has been reproduced in the simulation and can be seen in Figure 8 which shows the measured and simulated scour depths at 7 m, 10 m and 13 m from the inflow (the actual results were obtained from Wu, 2001). Figure 9 show that over time the smaller grains leave the flume and the bed slowly coarsens up. The bed reaches equilibrium when all the small grains are taken by the flow leaving only the coarse grains behind. This can also be seen in Figure 10 which shows the simulated and measured surface layer distribution at 10 m from the inlet.

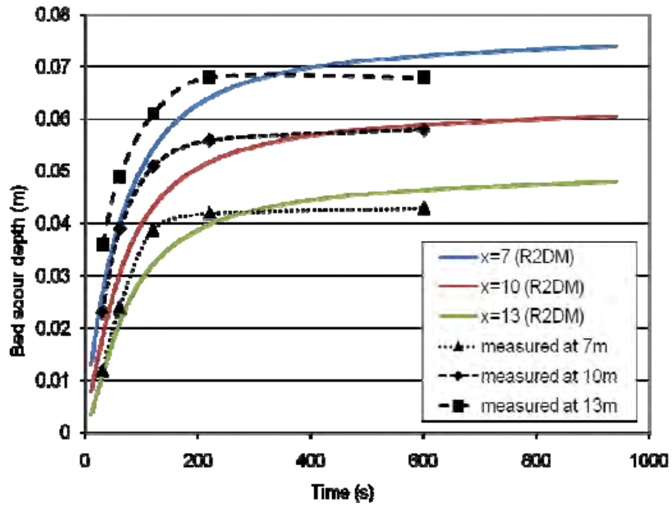

Figure 8. Simulated and measured scour depths for Ashida and Michiue (1971) experiment.

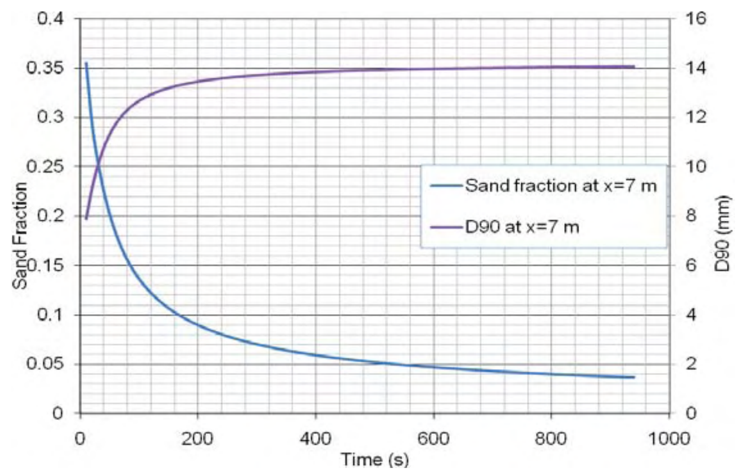

Figure 9. Simulated change in Sand fraction and D90 7 m from inlet.

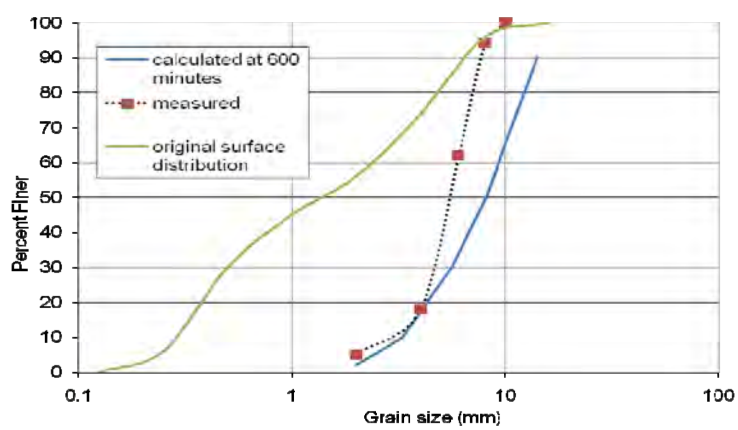

Figure 10. Simulated and measured surface layer distribution at 10 m from inlet at 600 minutes (after Wu, 2001).

There is good agreement between simulated and measured results for the bed changes. The surface layer distribution at 10 m from the inlet shows good agreement for grain sizes less than 5mm but simulated grain sizes in the upper range are approximately 1.5 times larger than measured.

Dispersion of Sediment Sediment often enters mountain rivers in discrete pulses associated with landslides and debris flows. R2DM was used to simulate a sediment pulse and the results were compared with Cui *et al.* (2003). Cui *et al.* (2003) simulated sediment pulses using a flume 45 m long, 0.5 m wide with a water discharge of 9 l/s, sediment feed rate of 45 g/min and slope of 0.0108. The following experimental procedure was adopted: 1) the bottom of the flume was covered with sediment; 2) the flume was allowed to reach a “pre-pulse” equilibrium; 3) the flow is temporarily halted so that a sediment pulse (3.5 cm high and 60 cm long for runs 2 and 3) could be formed at the upstream end of the flume; 4) the flow was started again and the flume was allowed to re-equilibrate.

The flume was discretized using a one dimensional mesh of 300 triangular elements with 302 nodes. The average distance between nodes was set to 0.3 m. It was not possible to simulate the same conditions as in the experiments because R2DM uses the same grain size distribution for the initial bed material throughout the computational domain. The simulation therefore used the same sediment mixture for the pulse and bed material upstream and downstream of it.

Results and Discussion of Sediment Dispersion Case Agreement with experimental results were obtained for run 2 using UW=0.9, C_{90} =5, k_T =0.1, L_s=0.01 and for run 3 using, C_{90} =5, k_T =0.1, L_s=0.01. The experimental observation in Figure 11 shows that the mode of pulse deformation is dispersive with a small degree of translation. The simulated results of Run 2 are in reasonable agreement with observed results. Better results of bed elevation could be obtained by increasing k_T but this increased sediment rates two orders of magnitude more than measured values.

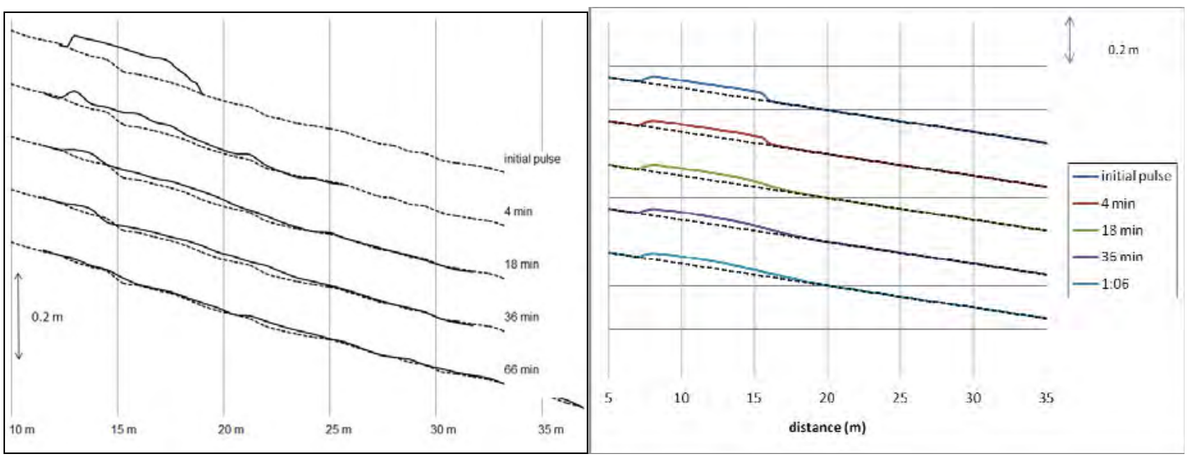

Figure 11. Observed and simulated long profile of average bed elevation for Run 2. Source: Cui *et al.*, 2003 (used with permission).

The results show that R2DM can capture the effects of dispersion of a sediment pulse. The results are not ideal but this was expected because of the following limitations of the model:

1. a different grain size distribution and for the sediment pulse and surrounding bed material could not be modeled; and
2. the thickness of the sediment pulse could not be specified;

The model over predicted transport rates by approximately one order of magnitude. This could be reduced by decreasing k_T but it would also have significantly reduced the changes to bed elevation.

Scour and Deposition in a Curved Flume There are very few published results of experiments that observe the effects of gravel transport in curved flumes and so it is not possible at this time to verify the 2D aspects of mixed size sediment transport with R2DM. Therefore a qualitative analysis of a simulation of a gravel bed in a curved flume will be compared to a simulation that uses a sandy bed for which experimental data exists.

The simulation uses the Laboratory of Fluid Mechanics (LFM) flume (Struiksma, 1985) to reproduce the results of Vasquez *et al.* (2005b). The flume was discretized using a mesh of 3831 elements with 2150 nodes. The bed elevation of both the inflow and outflow sections was kept constant and to minimize boundary effects the length of the straight sections were increased 10 m. The simulation was modeled using the Engelund-Hansen (Engelund and Hansen, 1967) sediment transport equation with D_{50}=0.78 mm, β=0.4 and *UW*=0. Secondary flow correction was computed using the described in Vasquez (2005b) but with r_c (in Equation [9] of Vasquez (2005b)) calculated using the actual streamlines and not from the velocity gradients.

Results and Discussion for Curved Flume Case The results of the sand bed and gravel bed simulations are shown in Figure 12. Since it appears that there is not much change in bed elevation for the gravel bed simulation, bed change for the gravel bed simulation is also shown. The results of both simulations are qualitatively similar but the sand bed simulation shows a greater bed change than the gravel bed simulation because the fine sediment (D_{50}=0.78 mm) is more mobile than gravel. The parameters used for the gravel bed simulation were: *UW* =0, k_T=1, L_s = 0.05, and C_{90}=2.5

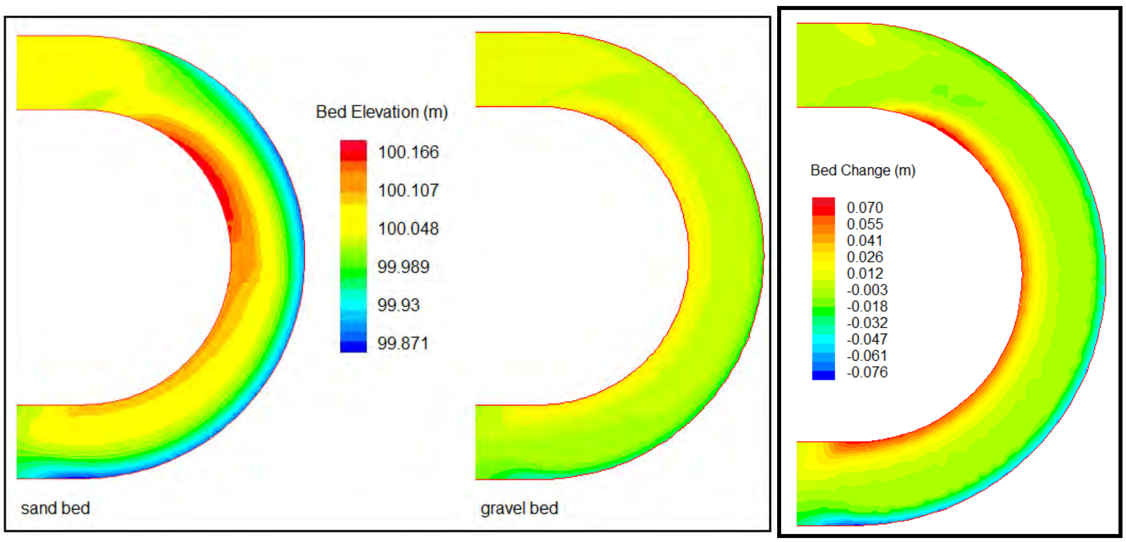

Figure 12. Scour and deposition in a curved flume for sand bed and gravel bed. Bed change shown for gravel bed simulation.

The results show that the model is correctly simulating the sediment transport. The curvature generates a secondary flow in the transverse direction from the outer bank, where scour occurs to the inner bank where a point bar forms. R2DM uses an algorithm to account for this secondary flow by making the sediment transport direction deviate from the depth-averaged flow velocity direction. This is shown in Figure 13 which shows the velocity vectors of the flow and bed-load transport.

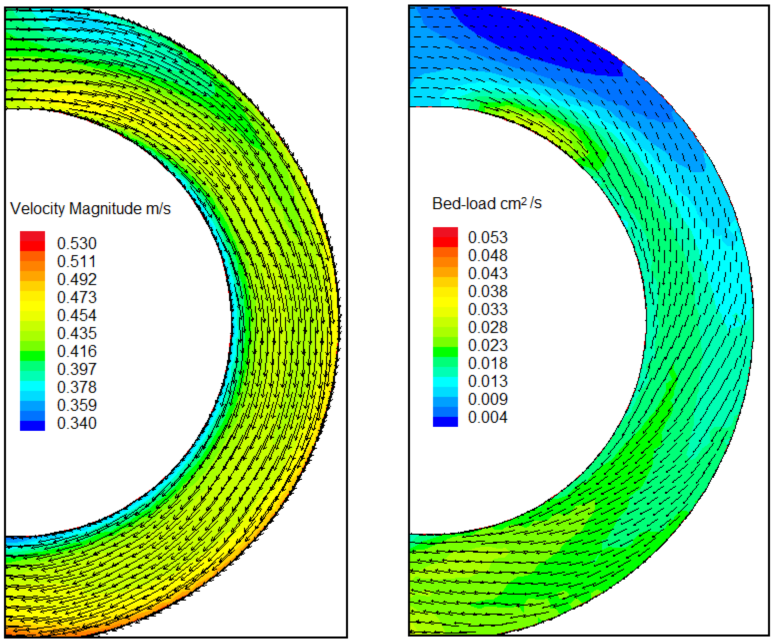

Figure 13. Velocity vectors of the flow and bed-load transport for gravel bed simulation.

Conclusion The river morphology model developed by Vasquez (2005) was upgraded to model mixed size sediment transport and migrating bed-forms. This model has been successfully applied to: straight alluvial channels to model bed aggradation, moving bed-forms, vertical sorting and armoring, bed degradation and sediment dispersion; and curved alluvial channels to capture sediment transport due to secondary flows. These features demonstrate that River2D Morphology can be potentially used to model a variety of different scenarios in natural rivers.

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