NUMERICAL MODELING OF LABORATORY FLUME EXPERIMENTS FOR TRACKING UNSTEADY SEDIMENT TRANSPORT USING COLORED PARTICLES

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Abstract: Developed at the National Center for Computational Hydrosience and Engineering, CCHE1D is a one dimensional numerical model for simulating unsteady flow hydrodynamics and sediment transport in natural dendritic channel systems, such as the drainage network of a watershed. CCHE1D has an Arc-GIS based user interface and can be run in coupled mode with AnnAGNPS and SWAT watershed models. The unsteady flow hydrodynamics uses a hybrid approach that solves full dynamic Saint-Venant equations when the flow is subcritical and reverts to a diffusive wave equation when the flow locally becomes transcritical or supercritical. The sediment transport and fluvial morphodynamics module calculates unsteady fractional sediment transport using a non-equilibrium sediment transport approach. The channel bed is modeled in three layers with the topmost layer being the mixing layer. The CCHE1D model calculates bed area change for each size class and tracks the percentage of each size class of sediment in the water column and in the upper two layers of the bed.

The CCHE1D model was used to model two laboratory experiments of sediment transport under unsteady flow conditions, which were carried out at the Hydraulic Research Laboratory of the Federal Institute of Technology in Lausanne, Switzerland. The experiments were performed in a 17.8m-long and 0.60m-wide tilting laboratory flume using symmetric triangular hydrographs with the same nominal base and peak discharge, but varying base widths. The mobile bed was made of gravel in the size range of 3–8mm, with a $d_{50}$ of 5.8mm. During the experiments, the discharge in the supply pipe, the water-surface elevation at eight stations, and the velocity profiles at three stations were measured. In addition, the submerged weight of the sediment particles falling into a sediment trap located near the downstream end of the flume was also continuously measured and recorded. To track the length of travel of sediment particles, the bed upstream of the sediment trap was prepared by laying eight strips of sediments with different colors. Each strip had a length of 0.7m and the total length of the reach with colored sediments was 5.6m. The colored sediments were used as tracers to track the movement during the passage of the triangular hydrograph. After an experiment with a triangular hydrograph, each colored sediment strip was divided into four strips of equal length. The sediment particles in each 0.175m-long strip were separated by hand into color groups and the groups were weighed. Similarly, the sediments collected in the sediment trap were also separated into color groups and weighed. The experimental results show that the length of travel is related to the width of the base of the hydrograph.

To be able to track the travel distances and locations of sediments of different color and size class in numerical simulations a total of 27 size classes were defined. This corresponds to nine colors, including the natural uncolored sediments, and the same three size classes for each color. The paper presents the results of two experiments, CM01 and CM02, and the corresponding numerical models. The computed results are compared with experimental measurements.
DESCRIPTION OF THE EXPERIMENTAL STUDY

The unsteady flow experiments over a movable bed were performed in a 17.75 m long tilting flume. The channel has a rectangular section which is 0.60m wide and 0.80m high. The side walls are made of transparent glass. The flume can be tilted to any slope from -1% to +9%. All the experiments described in the paper, were performed with the same bed slope of $S_o = 0.3\%$.

Figure 1 shows the general view of the experimental installation and the locations of the measuring instruments. The flume was operated in a closed circuit mode. A pump controlled by a computer pumps the water stored in tanks 1 and 2 into the head tank of the flume according to a prescribed triangular hydrograph. The pumped discharge is measured continuously by the electromagnetic current meter mounted on the supply pipe. Water entering the flume through a flow straightener flows over a 4.88m-long fixed-bed reach roughened by gluing artificial elements to the first 3.88m and fine gravel particles to the remaining 1.00m. Downstream of the fixed-bed section, there is a 9.22m-long and 10cm-deep movable-bed section filled with compacted fine gravel. A sediment trap is located at the downstream end of the movable-bed section. The perforated basket inside the sediment trap hangs from a force transducer, which continuously measures and records the cumulative weight of the sediment particles falling into the basket in order to determine the sediment discharge. The section of the flume downstream of the sediment trap has a fixed bed on
which the fine gravel particles are glued. At the downstream end of the flume, the flow freely falls into a collector and returns to storage tanks 1 and 2 to complete the circuit.

The movable bed is filled with fine gravel having a specific density of 2.65. Figure 2 shows the grain-size distribution curve obtained continuously using a video imaging technique. A few representative sizes are listed on the same figure. Considering that \(d_{95}/d_5 = 2.54 < 4\) and \(\sigma_g = \sqrt{d_{94.1}/d_{15.9}} = 1.33 < 1.35\), the bed material can be considered as uniform (Raudkivi 1988). Before each experiment the bed was raked, leveled, and compacted. Attention was paid to keep the grain-size distribution of the sediment the same for all experiments. The bed elevation along the flume centerline was measured and recorded before and after each experiment using an ultrasonic limnimeter mounted on a carriage (L8 in Figure 1). There was no sediment feed at the upstream of the movable bed. The bed-load transport rate during the experiment was low and the maximum bed level change at the end of experiments was found to be about \(d_{50}\).

![Grain size distribution curve](image)

**Figure 2** Grain size distribution curve of the fine gravel.

Figure 3 shows the longitudinal profile of the flume with the locations of the measuring stations and the layout of colored sediments used for bed-load tracking. The following variables were measured and recorded as a function of time:

- Discharge in the supply pipe was measured using an electromagnetic discharge meter.
- The flow depth were measured at eight locations along the flume length using ultrasonic limnimeters (L1 to L8 in Figure 3).
- Velocity profiles were measured at three locations along the flume (1, 2 and 3 in Figure 3) using six propeller current meters. At each measuring station two propeller current meters were positioned at prescribed elevations from the bed to measure the time varying local velocity. The same experiment was repeated 8 to 12 times by changing position of the propeller current meters. Time varying velocity profile over the entire flow depth was reconstituted from these point measurements. The velocity profiles were also then used to evaluate the time variation of shear stress, discharge and other useful flow parameters.
• The cumulative weight of the sediment particles falling into the basket in the sediment trap was recorded during the experiment. The bed-load transport rate at the end of the movable bed section was obtained from the cumulative weight curve. Given that the same experiment was repeated 8 to 12 times to obtain the complete velocity profile, it was possible to obtain a highly accurate measurement of time-varying bed-load rate by calculating the ensemble average.

Figure 3 Longitudinal profile of the flume showing locations of the measuring instruments and the layout of colored sediments used for tracking bed-load movement.

Figure 4 Definition sketch for the triangular hydrographs and the list of experiments.

Figure 4 shows the definition sketch for the triangular hydrograph and provides a list of the experiments considered in this paper. As it can be seen the experiments were performed with three different symmetric hydrographs ($T_r = T_f$) with different base times: 300s (TM01), 100s (TM03), and 20s (TM06, TM09 and TM10). The base and the maximum discharges for the hydrographs are nominally same to each other. The unsteadiness parameter, $\Omega = \left(\frac{h}{u^*}\right) (\Delta Q/\Delta t)$, for the experiments is also listed on the last column of the table in Figure 4. After the experiments with three hydrographs were completed, one additional experiment was performed with each hydrograph using colored sediments for tracking the bed load. These experiments are listed in Figure 4 as CM01 (300s), CM02 (100s), and CM03 (20s).
The colored sediments were produced by painting the fine gravel used as bed material and have the grain-size distribution shown in Figure 2. The gravel was immersed in paint and then dried on plastic sheets. Attention was paid to keep the paint thin in order not to change the characteristics of the particles. Eight different colors were used Black (B), Golden (G), Red (R), Green (E), Argent (A), Blue (U), Yellow (Y), and Copper (C). Natural, uncolored sediments are denoted by “N”. Before each bed-load tracking experiment, the colored sediments were carefully laid on top of the natural colored sediments in 0.70m-long stripes along the 5.6m reach immediately upstream of the sediment trap as shown in Figure 3. The thickness of the bed layer was about 3 cm (~5×d50).

![Figure 5 Appearance of the bed with colored sediments at the end of the experiments.](image)

There is no sediment transport during the base flow. As the discharge increases, the bed shear increases above the critical shear stress to initiate the movement. The sediments are transported downstream as bed-load. The general appearances of the bed at the end of experiments CM01, CM02 and CM03 are shown in Figure 5. In order to determine how far the sediments traveled during the passage of the hydrograph, each color reach was divided into four equal length stripes of 17.5cm. The sediment particles in the 3cm-deep surface layer were collected carefully. The sediments collected from each stripe were then manually separated into different colors, including the natural sediments that were transported from upstream. Dry weight of sediments in each color were then measured and recorded.

The unsteady flow experiments discussed in this paper are part of an extensive experimental study performed at the Laboratory of Environmental Hydraulics of the Federal Institute of Technology, Lausanne, Switzerland. Detailed information on the complete set of experiments can be found in Qu (2002). A short presentation of the experimental study can be found in Qu et al. (2004). A preliminary analysis of the experiment with colored sediments is presented in Altinakar et al. (2014).

**BRIEF DESCRIPTION OF THE CCHE1D NUMERICAL MODEL**

The experiments CM01 and CM02 with colored sediments were simulated using the CCHE1D software, which was developed at the National Center for Computational Hydrosceince and
CCHE1D computes unsteady flows by solving the Saint-Venant equations given as:

\[
\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q
\]  

(1)

\[
\frac{\partial}{\partial t} \left( \frac{Q}{A} \right) + \frac{\partial}{\partial x} \left( \beta \frac{Q^2}{2A^2} \right) + g \frac{\partial h}{\partial x} + g(S_f - S_o) = 0
\]

(2)

where \(x\) and \(t\) are the spatial and temporal axes; \(A\) is the flow area; \(Q\) is the flow discharge; \(h\) is the flow depth; \(S_0\) is the bed slope; \(\beta\) is a correction coefficient for the momentum due to the non-uniformity of velocity distribution at the cross section; \(g\) is the gravitational acceleration; and \(q\) is the side discharge per unit channel length. \(S_f\) is the friction slope, defined as:

\[
S_f = \frac{Q|Q|}{K^2} \quad \text{with} \quad K = \sum_{l=1}^{3} \frac{1}{n_l} A_l R_l^{2/3} = \sum_{l=1}^{3} \frac{1}{n_l} A_l^{5/3} P_l^{-2/3}
\]

(3)

where \(A_l\) is the flow area of subsection \(l\); \(R_l\) is the hydraulic radius; \(P_l\) is the wetted perimeter; and \(n_l\) is the Manning’s roughness coefficient. \(l\) is the index for subsections 1, 2 and 3 (left flood plain, main channel and right flood plain, respectively). CCHE1D employs an improved version of the implicit four-point finite difference scheme proposed by Preissmann (1961) to discretize the Saint-Venant equations. The solution scheme uses the increments in water depth \(\Delta h\) and flow discharge \(\Delta Q\) as dependent variables. The resulting system of equations form a pentadiagonal matrix, whose solution is obtained with the help of the Thomas Algorithm, also called the Double-Sweep Method. CCHE1D adopts a hybrid dynamic/diffusive wave model. The dynamic wave model is solved when the Froude number is less than 0.9. For higher Froude numbers, CCHE1D solves the diffusive wave model.

The governing equations for the 1D non-equilibrium transport of non-uniform sediment and the resulting bed area change are given by the following expressions, respectively

\[
\frac{\partial (AC_{tk})}{\partial t} + \frac{\partial Q_{tk}}{\partial x} + \frac{1}{L_b} (Q_{tk} - Q_{t^*k}) = q_{tk}
\]

(4)

\[
(1 - p') \frac{\partial A_{bk}}{\partial t} = \frac{1}{L_b} (Q_{tk} - Q_{t^*k})
\]

(5)

where \(C_{tk}\) is the section-averaged sediment concentration of size class \(k\); \(Q_{tk}\) is the actual sediment transport rate of size class \(k\); \(Q_{t^*k}\) is the sediment transport capacity of size class \(k\); \(L_b\) is the
adaptation length of non-equilibrium sediment transport; and $q_{lk}$ is the side inflow or outflow sediment discharge from bank boundaries or tributary streams per unit channel length, $p'$ is the porosity of the bed material, and $\partial A_{bk}/\partial t$ represents the rate of change of bed area for the size class $k$. The first term on the left-hand side of Eq. (4) accounts for sediment storage, while the last term on the left-hand side represents the exchanges between the moving sediment and the bed material. The transport capacity for size class $k$ can be expressed as:

$$Q_{t^*k} = p_{bk} Q_{tk}$$

where $p_{bk}$ is the percentage of sediments of size class $k$ in the bed material. The potential equilibrium transport rate for sediments of size, $Q_{tk^*}$, can be determined using a suitable empirical sediment transport formula. To take into account the variation of the vertical gradation of sediment size due to erosion and deposition processes, the bed is represented in three layers. The topmost layer is the mixing layer. All sediment particles in the mixing layer are subject to exchange with the water column. The second layer is the subsurface layer. The subsurface layer exchanges sediments only with the mixing layer. There is no sediment exchange between the subsurface layer and the bottommost third layer. The temporal variation of the bed-material gradation in the mixing and subsurface layers is determined using the following relationships, respectively:

$$\frac{\partial (A_m p_{bk})}{\partial t} = \frac{\partial A_{bk}}{\partial t} + p_{bk}^* \left( \frac{\partial A_m}{\partial t} - \frac{\partial A_b}{\partial t} \right)$$

$$\frac{\partial (A_s p_{sbk})}{\partial t} = -p_{bk}^* \left( \frac{\partial A_m}{\partial t} - \frac{\partial A_b}{\partial t} \right)$$

where $A_m$ and $A_s$ are the bed areas for the mixing and subsurface layers, respectively, $A_b$ is the total bed area. The total bed deformation rate is the sum of bed deformation rates of all size classes, i.e. $\partial A_b/\partial t = \sum_{k=1}^{N} \partial A_{bk}/\partial t$ in which $N$ is the number of size classes used. When $(\partial A_m/\partial t - \partial A_b/\partial t) \leq 0$ one has $p_{bk}^* = p_{bk}$, otherwise $p_{bk}^* = p_{sbk}$. This system of equations is closed by introducing semi-empirical expressions for various parameters, such as non-equilibrium adaptation length, $L_b$, thickness of mixing layer, sediment porosity, settling velocity of grains, etc. The system of equations is also discretized and solved using Preissmann’s (1961) method.

SIMULATION OF EXPERIMENTS USING CCHE1D AND RESULTS

CCHE1D was used to simulate the experimental test cases CM01 and CM02 by dividing the grain size distribution curve shown in Figure 2 into three size classes: 1) coarse and very coarse sand (0.5mm<d<2mm); 2) fine and very fine gravel (2mm<d<8mm); and 3) medium gravel (8mm<d<16mm) where d is the sediment diameter. Furthermore, in order to be able to track the sediments with different colors during the simulation, the same size distributions were repeated for each of the eight colors as well as the natural sediments. This resulted in 27 size classes as shown in Table 1. All size classes have a dry specific density of 2.65.

The computational mesh comprises 336 grid nodes with a fixed grid spacing equal to 5 cm. The Manning roughness is calibrated as 0.016 m$^{-1/3}$/s and the time step is 0.5 second. The thickness of mixing layer is assumed to be twice the mean grain size. Wu et al. (2000) formula is used to
compute the transport capacity. The characteristics of the triangular hydrograph imposed at the upstream end of the model for CM01 and CM02 simulations are summarized in Figure 4.

Table 1 Sediment size classes used for tagging the sediments of different color. All sediments have a dry specific density of 2.65.

| ID | d_{rep} (mm) | D_{LL} (mm) | D_{UL} (mm) | Color       | ID | d_{rep} (mm) | D_{LL} (mm) | D_{UL} (mm) | Color       |
|----|-------------|-------------|-------------|-------------|----|-------------|-------------|-------------|-------------|-------------|
| 1  | 1.00        | 0.50        | 2.00        | Natural (N) | 16 | 1.00        | 0.50        | 2.00        | Green (E)   |
| 2  | 4.00        | 2.00        | 8.00        | Copper (C)  | 17 | 4.00        | 2.00        | 8.00        | Red (R)     |
| 3  | 11.31       | 8.00        | 16.00       | Yellow (Y)  | 18 | 11.31       | 8.00        | 16.00       | Golden (G)  |
| 4  | 1.00        | 0.50        | 2.00        | Blue (U)    | 19 | 1.00        | 0.50        | 2.00        | Black (B)   |
| 5  | 4.00        | 2.00        | 8.00        |             | 20 | 4.00        | 2.00        | 8.00        |             |
| 6  | 11.31       | 8.00        | 16.00       |             | 21 | 11.31       | 8.00        | 16.00       |             |
| 7  | 1.00        | 0.50        | 2.00        | Silver (S)  | 22 | 1.00        | 0.50        | 2.00        |             |
| 8  | 4.00        | 2.00        | 8.00        |             | 23 | 4.00        | 2.00        | 8.00        |             |
| 9  | 11.31       | 8.00        | 16.00       |             | 24 | 11.31       | 8.00        | 16.00       |             |
| 10 | 1.00        | 0.50        | 2.00        |             | 25 | 1.00        | 0.50        | 2.00        |             |
| 11 | 4.00        | 2.00        | 8.00        |             | 26 | 4.00        | 2.00        | 8.00        |             |
| 12 | 11.31       | 8.00        | 16.00       |             | 27 | 11.31       | 8.00        | 16.00       |             |
| 13 | 1.00        | 0.50        | 2.00        |             |    |             |             |             |             |
| 14 | 4.00        | 2.00        | 8.00        |             |    |             |             |             |             |
| 15 | 11.31       | 8.00        | 16.00       |             |    |             |             |             |             |

d_{rep} = representative diameter of a size class; D_{LL} = lower limit of a size class; D_{UL} = upper limit of a size class.

In Figure 6 and Figure 7, the water surface elevations computed at the locations of the eight acoustic limnimeters (L1 to L8 in Figure 3) are compared with the measurements for experiments CM01 and CM02, respectively. The agreement is relatively good although the computed hydrographs have systematically higher peaks. Since the travel distance of the wave over the length of the flume is short, the attenuation is not important and the wave keeps its shape. In Figure 8 and Figure 9, the discharge, velocity and flow depth computed at the three measuring stations, “D” (x=14.07m), “M” (x=11.07m), and “U” (x=5.10m), are compared with the measurements for experiments CM01, and CM02, respectively. For the experiment CM01 with slower rising and falling hydrograph, the computed discharge and depth shown in Figure 8 agree well with the measurements at all three stations. The computed velocity, however, is larger than the measured one for the stations “D” and “M”. In Figure 9, similar plots are provided for the experiment CM02 with a faster rising and falling hydrograph. The measurements at the station “U” were not recorded due to a faulty cable. Although the computed flow depths agree well with the measurements at stations “D” and “M”, the computed discharge and velocity are under predicted. This may be due to the secondary effects introduced by the faster hydrograph that are not well modeled with the Saint-Venant equations.

The CCHE1D continuously tracks the percentage of each size class in the bed. Since the ID numbers of three size classes representing each color, including the natural color, are known, it was possible to calculate the weight of each color as a function of the distance. This in turn gives the information about the travel distance of each size class during the passage of the triangular hydrograph.
The computed mass of the sediments of different colors composing the bed after the passage of the triangular hydrographs in CM01 and CM02 are plotted in Figure 10 and Figure 11, respectively. The experimental data are superposed on the computed weights. Since the black sediments are falling immediately into the basket of the sediment trap, the rightmost plot at the bottom row shows a histogram of the mass of the sediment of different colors accumulated in the basket. The agreement between the computed and measured sediment weights as a function of the distance is satisfactory despite some differences that are probably well within the uncertainty to be expected in comparison with a single sediment transport experiment. For the experiment CM01, the computed mass of the sediment accumulated in the basket is 1,609 gr. A series of 12 experiments repeated with the same hydrograph as in CM01 yielded the average mass of the sediments as 1190.63 gr ± 216.60 gr. The computed value is about 200 gr higher than the upper limit of experiments. Similarly, the computed mass of sediments falling into the basket is 698.65 gr, whereas the experimental results a series of 12 experiments with the same hydrograph as in CM02 yields and average mass of 832.31 gr ± 216.87 gr. The computed value is well within the experimental range. Based on these results it can be concluded that the simulations of experiments with CCHE1D produce results that are comparable to those measured in the experiments.
Figure 8 Comparison of measured and computed discharge, velocity and flow depth at three measuring stations D (x=14.07m), M (x=11.07m), and U (x=5.10m) for the experiment CM01.

Figure 9 Comparison of measured and computed discharge, velocity and flow depth at three measuring stations D (x=14.07m), M (x=11.07m), and U (x=5.10m) for the experiment CM02.
Figure 10 Comparison of computed and measured travel distances of the sediments for CM01.

Figure 11 Comparison of computed and measured travel distances of the sediments for CM02.
CONCLUSIONS

The CCHE1D model developed at the national Center for Computational Hydroscience and Engineering was used to simulate two out of three experiments carried out at the Laboratory of Environmental Hydraulics of the Federal Institute of Technology, Lausanne, Switzerland, for tracking the movement of colored sediments during the passage of a triangular hydrograph. The hydrograph for CM01 rises from 50 m$^3$/s to 140 m$^3$/s in 150s and falls back to same base discharge in 150s. The hydrograph for CM02 rises from 40 m$^3$/s to 160 m$^3$/s in 50s and falls back to same base discharge in 50s. CM02 has a faster rising and falling hydrograph than CM01 for the same nominal base and peak discharges. The unsteadiness parameter, $\Omega = (h/\text{\textit{u}}_*^2)(\Delta Q/\Delta t)$, where $\text{\textit{u}}_*$ is the shear velocity and $\Delta Q = Q_m - Q_b$, is 0.41 for CM01 and 1.64 for CM02. Comparison of computed results with the experimental results show that the computed water surface elevations, discharges and velocities agree well with the measurements. For the experiment CM02, the velocities are under predicted although discharge and flow depth show better agreement. This may be due to secondary effects that are not well represented by the Saint-Venant equations. The computed distances traveled by each color agree well with the experimental results. The computed mass of the sediments accumulated in the basket of the sediment trap located at the downstream end of the movable bed section also agree well with the experimental data. Additional simulations by varying various parameters such as the adaptation length and the mixing length thickness, are currently under way. Moreover, the simulations are also being carried out for the third colored experiment, which rises from 40 m$^3$/s to 140 m$^3$/s in only 10s and falls back to same base discharge again in 10s. This experiment has the fastest hydrograph and the unsteadiness parameter is about 6.9, which is significantly higher than those for CM01 and CM02. The results of the ongoing simulations will be reported during the conference.

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