TWO-DIMENSIONAL NUMERICAL MODELLING OF HYPERCONCENTRATED FLOWS

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Abstract: SRH-2D, a two-dimensional sediment transport model developed at the U.S. Bureau of Reclamation, is extended to simulate hyperconcentrated flows by considering the effect of sediment concentration on the the sediment fall velocity, sediment capacity and sediment erosion and deposition. The governing equations for shallow water equations are modified to be suitable for two dimensional unsteady hyperconcentrated flow modeling. Density variance and sediment exchanges between flow and bed are incorporated. The revised model is tested and verified using two cases: an idealized transient flow and sediment process due to a dam-break flow over a horizontal bed and a more complex dam-break flow with a sudden width enlargement. The model results show that the numerical model can predict the water surface elevation with reasonable accuracy. The results also show that the extra terms related to the density gradient and the terms related to the exchange between the flow and the erodible bed are important for hyperconcentrated flow. They increase the water front speed and wave height during a dam break.

INTRODUCTION

Hyperconcentrated flow can occur in high-discharge flows of water and sediment. The definition of hyperconcentrated flow varies among different researchers. Pierson (2005) defines hyperconcentrated flows to be between two extremes. At the low extreme, water transports sediment in such quantities that the sediment has negligible effect on flow behavior and the fluid remains Newtonian. At the other extreme, high-discharge debris flows and mudflows transport much more sediment than water can carry. For such cases sediment concentrations are often in excess of 60% by volume and 80% by weight. For debris and mud flows sediment plays a very important role in flow behaviors and mechanics (Wan and Wang, 1994; Coussot and Piau, 1994). Some researchers refer the debris flows and mudflows also as hyperconcentrated flow (NRC, 1982; O'Brien and Julien, 1985; and Julien and Paris, 2010).

One dimensional (1D) numerical modelling has often been used to simulate hyperconcentrated sediment-laden flows. For example, Zhang et al. (2001) presented a 1D unsteady numerical model for hyperconcentrated sediment-laden flows and applied it to the Lower Yellow River with a length of 394km. A semi-empirical equation of sediment transport capacity was used. Cao et al. (2004 and 2006) presented a 1D numerical model for a hyperconcentrted sediment laden flow. They used a second-order TVD method in conjunction with the HLLC Riemann solver and the SUPERBEE limiter. The extra terms related to the mixture density were included in the momentum equation, but not in the mass conservation equation. The exchange between the flow and the erodible bed was included in the mass and momentum equations. Wu and Wang (2007) presented a 1D hyperconcentrated model for dam-break flow over movable beds. The mass and momentum equations included the contribution of the sediment erosion and deposition. Guo et al. (2008) presented a hyper-concentrated sediment transport model for the lower Yellow River. The model considered the effect of high concentration on the flow density and sediment settling velocity. The variation of the mixture density and the exchange between the flow and the erodible bed were not included in the mass and momentum equations. Mouri et al. (2011) simulated an extreme flood and sediment flux in a mountain stream with a 1D numerical model. The mass and momentum equations considered no contribution of the sediment erosion and deposition. Abderrezzak and Paquier (2011) investigated sediment transport capacity formulas in dam-break flows. He et al. (2012) simulated unsteady hyperconcentrated sediment-laden flow in the Yellow River, China. They incorporated the extra terms related to the density of turbid flow into mass and momentum conservation equations, but not the contribution of sediment exchange between the flow and the bed.

Kim and Lee (2012) presented a 1D and 2D finite volume method for a hyperconcentrated sediment laden flow in a Cartesian coordinate system. They pointed out that the contribution of density should not be ignored in a hyperconcentrated sediment laden flow. Shallow water equations (SWE) with variable density were solved and the

sediment erosion and deposition terms were taken account in the depth-averaged continuity and momentum equations.

In this paper, we develop a 2D hyper-concentrated flow and sediment model based on the existing SRH-2D model and previous 1D model studies. .

DESCRIPTION OF THE NUMERICAL MODELS

The governing equations for 2D, unsteady, hyperconcentrated sediment-laden flow were revised from shallow water equation to include the density variance and sediment exchanges between flow and bed. The governing equations include mass and momentum conservation equations for the water-sediment mixture, the mass conservation equations for each sediment class carried in the flow, and the bed morphological change equation.

$$\frac{\partial(h)}{\partial t}+\frac{\partial(hU)}{\partial x}+\frac{\partial(hV)}{\partial y}=\frac{(E-D)}{1-P} \tag{1}$$

$$\frac{\partial(hU)}{\partial t}+\frac{\partial(hU^2)}{\partial x}+\frac{\partial(hUV)}{\partial y}=-gh\frac{\partial z_s}{\partial x}-\frac{\tau_{bx}}{\rho}-\frac{\rho_s-\rho_m}{2\rho_s}gh^2\frac{\partial S_v}{\partial x}-\frac{\rho_b-\rho_m}{\rho_m}\frac{(E-D)U}{1-P} \tag{2}$$

$$\frac{\partial(hV)}{\partial t}+\frac{\partial(hUV)}{\partial x}+\frac{\partial(hV^2)}{\partial y}=-gh\frac{\partial z_s}{\partial y}-\frac{\tau_{by}}{\rho}-\frac{\rho_s-\rho_m}{2\rho_s}gh^2\frac{\partial S_v}{\partial y}-\frac{\rho_b-\rho_m}{\rho_m}\frac{(E-D)V}{1-P} \tag{3}$$

$$\frac{\partial(hS_k)}{\partial t}+\frac{\partial(hUS_k)}{\partial x}+\frac{\partial(hVS_k)}{\partial y}=E_k-D_k \tag{4}$$

$$(1-p)\frac{\partial z}{\partial t}=\sum_k(D_k-E_k) \tag{5}$$

where t = time (s); x and y=horizontal Cartesian coordinates (m); h = water depth (m); U and V = depth-averaged velocity components (m/s) in x and y directions, respectively; g = gravitational acceleration; z_s is water surface elevation (m); z is bed elevation (m); $\rho_m = \rho_0(1-S_v)+\rho_s S_v$=density of the water-sediment mixture (kg/m^3); $\rho_b = \rho_0 p+\rho_s(1-p)$=density of the saturated bed (kg/m^3); ρ_0 and ρ_s= densities of water and sediment (kg/m^3); respectively; S_v=$\sum S_k$= total volumetric sediment concentration; S_k = volumetric sediment concentration of size class k; p = bed sediment porosity; E and D = total sediment entrainment and deposition fluxes (m/s) across the bottom boundary of flow, representing the sediment exchange between the water column and the bed. E_k and D_k = sediment entrainment and deposition fluxes of size class k; and bed friction is calculated using the Manning's roughness equation as follows:

$$\begin{pmatrix}\tau_{bx}\\ \tau_{by}\end{pmatrix}=\rho C_f\begin{pmatrix}U\\ V\end{pmatrix}\sqrt{U^2+V^2};\qquad C_f=\frac{gn^2}{h^{\frac{1}{3}}} \tag{6}$$

where n is the Manning's roughness coefficient.

There are two ways to calculate sediment erosion and deposition for superconcentrated flows.

Cao et al. (2004 and 2006) presented sediment erosion and deposition for superconcentrated flows as,

$$D=\alpha c\omega_0(1-\alpha c)^m \tag{7}$$

$$E=\begin{cases}\varphi(\theta-\theta_c)uh^{-1}d^{-0.2} & \text{if } \theta\geq\theta_c\\ 0 & \text{else}\end{cases} \tag{8}$$

where D, E=sediment deposition and entrainment fluxes across the bottom boundary of flow (m/s), representing the sediment exchange between the water column and bed; ω_0= settling velocity in clear water (m/s), α is specified empirically by $\alpha = \min[2, \frac{1-p}{c}]$; m = exponent=2.0; h = flow depth (m); d = sediment particle diameter (m) of size class k; p = bed sediment porosity (-) =0.4; $R = \sqrt{sgd}d/\nu$; $s = \frac{\rho_s}{\rho_0} - 1$; ν = kinematic viscosity of water = 1.2E-6 m^2/s; and θ = Shields parameter=$U_*^2/(sgd)$; θ_c = critical Shields parameter for initiation of sediment movement = 0.045; and U_*=shear velocity (m/s);c is depth-averaged volumetric sediment concentration; and φ=0.015[$m^{1.2}$].

Another method is to estimate the sediment deposition and erosion the same way as in other low concentration sediment transport numerical models. The sediment transport capacity for hyperconcentrated flow is presented in the previous section. Sediment erosion and deposition for the kth-sized sediment group were calculated based on sediment transport capacity $S_{*,k}$ and volumetric sediment concentration S_k, respectively.

$$D = \alpha \omega_{s,k} S_k \tag{9}$$

$$E = \alpha \omega_{s,k} S_{*,k} \tag{10}$$

where α=recovery coefficient which ranges from 0.001 in the case of continuous deposition to 1.0 for severe erosion; $\omega_{s,k}$= settling velocity for kth-sized sediment group (m/s). Compared with Eqs. () and (), it is found that the recovery coefficient α is the ratio between the bottom and average concentrations.

Many equations have been proposed to calculate the sediment transport capacity (Zhang and Zhang, 1992; Guo et al., 1995; Yang et al., 1996; Ni et al., 2004; He et al., 2012, and Taiwan Water Resources Agency, 2012). Due to space limit, only two sediment transport capacity equations are presented here which are used in the second test case.

Wu and Long (1993) presented suspended sediment transport capacity, written as

$$S_{*k} = k\left[\frac{\rho_m}{\rho_s - \rho_m}\frac{u^3}{gh\omega_m}\right]^{\mathrm{M}} \Delta P_{*k} \tag{11}$$

where S_{*k}= suspended sediment concentration of class k (kg/m^3); k and M are two empirical parameters, k = 0.452kg/m^3 and M = 0.762; h and u = averaged depth (m) and velocity (m/s) in a cross section, respectively; ω_m = group settling velocity = $\left(\sum_{k=1}^{N_s} \Delta P_{*k}\omega_{sk}^M\right)^{1/M}$; ω_{sk} = settling velocity of sediment size k; ΔP_{*k} = percentage of suspended sediment transport capacity for the k-th class = $(\Delta P_{bk}/\omega_{sk})^\phi / \sum_{k=1}^{N_s}(\Delta P_{bk}/\omega_{sk})^\phi$; ΔP_{bk} =bed material size fraction for the k-th class; ϕ is an empirical coefficient = 0.8; ρ_s and ρ_m are sediment and flow sediment mixture densities (kg/m^3), respectively; g = gravitational acceleration.

A formula proposed by Dou et al. (1999) can be used to determine the bed-load transport capacity, which is written as

$$q_{\mathrm{b}k} = \frac{K_b}{C_0^2}\frac{\rho_s\rho_m}{\rho_s - \rho_m}(u - u_{ck})\frac{u^3}{g\omega_m}\Delta P_{bk} \tag{12}$$

where $q_{\mathrm{b}k}$ = bed-load transport capacity per unit width for the k th grain size fraction (kg/ms); K_b = empirical coefficient (=0.1 calibrated by experiment); C_0 = dimensionless Chezy coefficient = $C\sqrt{g}$ in which $C = h^{1/6}/n$ and n = Manning's coefficient; and u_{ck} = incipient velocity (m/s) of the k-th bed-load fraction; calculated by

$$u_{ck} = \left(\frac{h}{d_k}\right)^{0.14}\sqrt{17.6\frac{\rho_s - \rho_0}{\rho_0}d + 6.05 \times 10^{-7} \times \frac{10 + h}{d_k^{0.72}}} \tag{13}$$

where h = water depth (m); d_k = grain diameter (m) of class k; ρ_s and ρ_o are sediment and clear water densities (kg/m^3), respectively.

The 2D flow and sediment transport model, SRH-2D, is updated to include the extra terms related to the density gradient and the exchange between the flow and the erodible bed in the mass and momentum equations (Eqs. through). In the mass conservation equation (Eq.), the RHS represents the mass exchange between the flow and the erodible bed, which might be potentially signification in a hyperconcentrated flow which large erosion or deposition. The 3rd terms in the RHS of Eqs. (and) represent the effect of density variations, which makes the current hyperbolic system differ from the traditional shallow water equation in which the sediment concentration has no impact on the flow. The 4th terms in the RHS of Eqs. () and () represent the momentum transfer due to sediment exchange between the water and the erodible bed.

MODEL TESTS AND VERIFICATIONS

Simulation of a Dam-break Flow in a Mobile Channel: The numerical model is used to simulate an idealized transient process due to a dam-break flow over a horizontal bed. The channel length is set to be 50km, and the dam is initially located at the middle of the channel (x=25 km). The initial water depths upstream and downstream of the dam are h1=40 m and h2=2 m, respectively. Uniform sediment of size d=4 mm is used and the Manning's roughness of n=0.03 is presumed.

To simulate the 1D problem, three cells are used in the transversal direction and symmetric boundary condition is used for the two side boundaries. The simulation is focused on the transient process due to dam-break, rather than process driven by unsteady hydrographs, thus the wall boundary condition is adopted for the upstream and downstream boundaries. A spatial step of $\Delta x = 10$ m and a time step of 0.1 s are used. Initial sediment concentration was unknown in the original paper by Cao et al. (2004), and zero initial sediment concentration is chosen here.

Figure 1 shows a comparison of the free surface and bed profiles simulate by SRH-2D with and without those extra terms associated with hyperconcentrated flows and in Cao et al. (2004). The following are observed.

Bed erosion occurs during dam break near the location of dam. Bed erosion is a combined effect of sediment eroded from the bed and deposited on the bed. SRH-2D obtained similar bed erosion with Cao et al. (2004). The location of the maximum erosion originally occurs at the dam, and then moves upstream.

Ignoring the extra terms related to the density gradient and the terms related to the exchange between the flow and the erodible bed increases the bed erosion during a dam break problem. It can be understood that the extra term from the sediment eroded from the bed would slow down the momentum of the flow, thus reduce the capability of the flow to erode bed. However, this observation is only valid in a horizontal channel without slope. The increased sediment density could increase the channel's capacity to erode bed in a sloped channel.

The extra terms related to the density gradient and the terms related to the exchange between the flow and the erodible bed in hyperconcentrated flow increase the wave front speed and height.

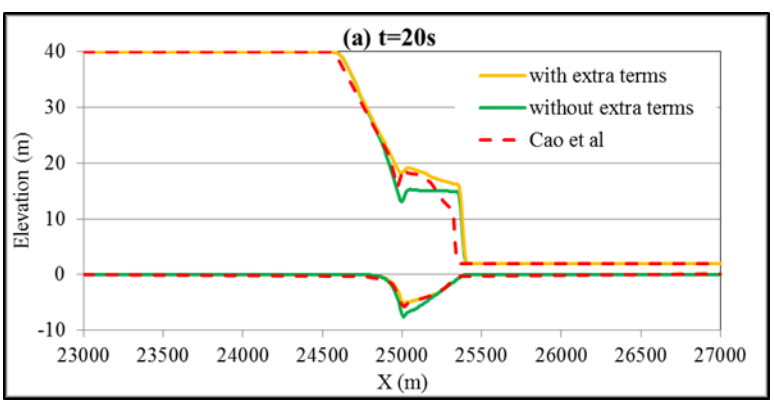

Figure 1 Evolution of water surface and bed profiles

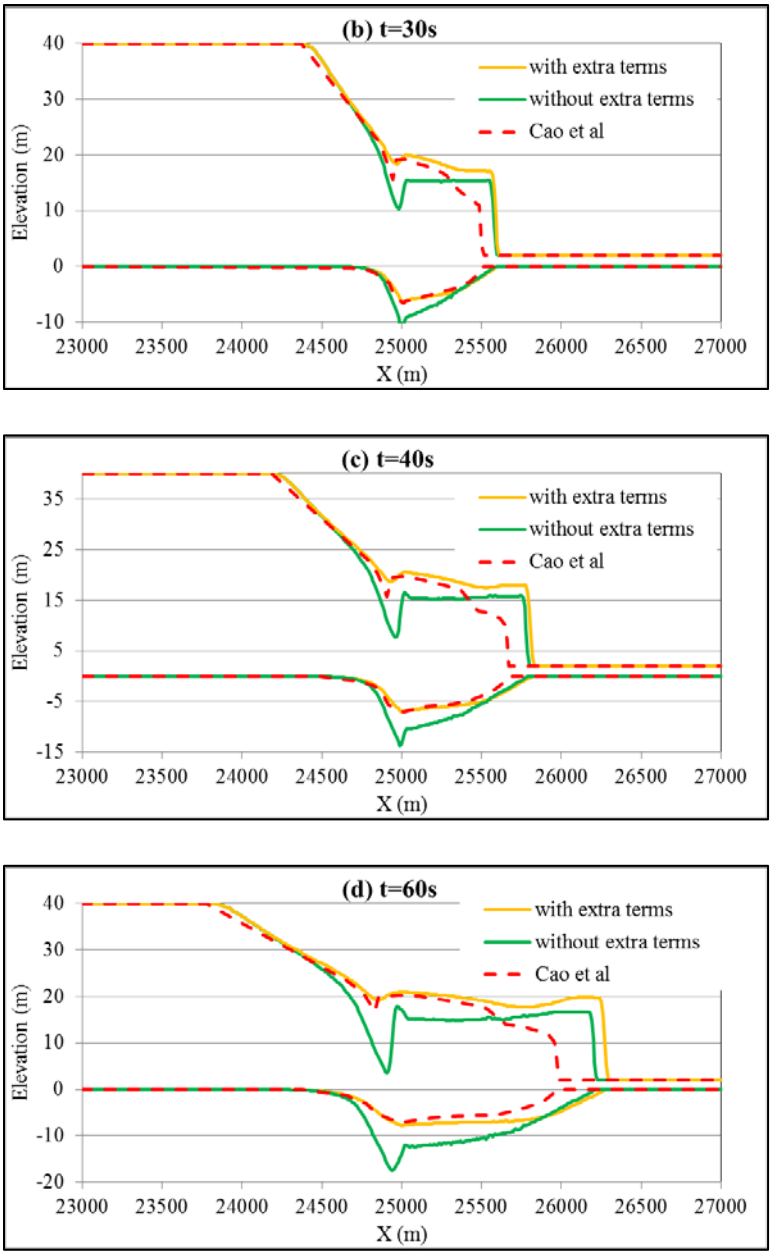

Figure 1 Continued

Simulation of a Dam-break Flow in a Mobile Channel with a Sudden Enlargement: Dam break experiments over mobile beds were performed at the Civil Engineering laboratory of the Université Catholique de Louvain, Belgium (Zech et al., 2008; Spinewine and Zech, 2007; and Palumbo et al., 2008) and was simulated by Xia et al. (2010).

The experiment was conducted in a 6m long flume with a non-symmetrical sudden enlargement from 0.25 m to 0.5 m width located 1 m downstream for the gate at x=3 m. The dam break was simulated by a rapid downwards movement of a thin gate at the middle of the flume to reach initial test conditions that approach as much as possible the idealization of an instantaneous dam collapse while minimizing perturbations to the sediment and water during gate removal. The sediment used was uniform coarse sand ranging from 1.2 to 2.4 mm with d_{50}=1.82 mm and a density of 2680 kg/m^3. Before bed profiling, the sand was compacted in place to reach a reproducible solid packing of concentration 53% (47% porosity). The initial conditions included 0.1 m sand thickness over the whole flume

and an initial clear water depth of 0.25 m upstream of the gate and dry bed downstream of the bed. A Manning's coefficient of 0.026 was used for the simulation.

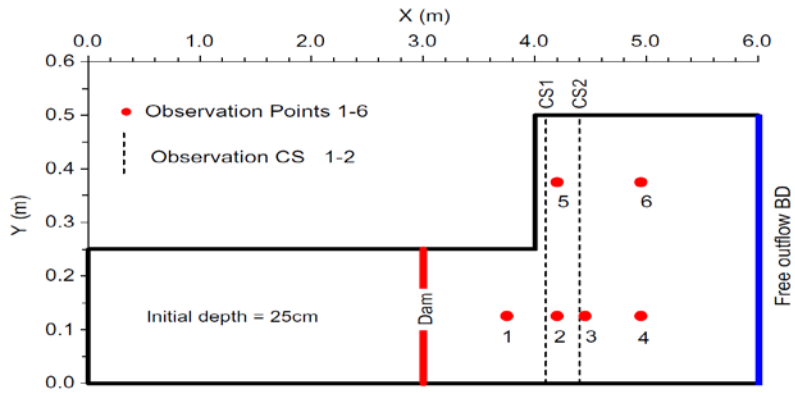

Figure 2 Sketch of a dam-break flow experiment over a mobile bed (Source: Xia et al., 2010)

Figure 3 shows measure and simulated water surface elevation time series, Xia et al. (2010) simulated, and SRH-2D simulated at locations P1 (3.7, 0.125), P2 (4.2,0.125), P3 (4.45,0.125), P4 (5.0, 0.125), P5 (4.2, 0.375), and P6 (5.0, 0.375). It can be seen that the current numerical model predicts the water surface elevation well in both the magnitude and timing.

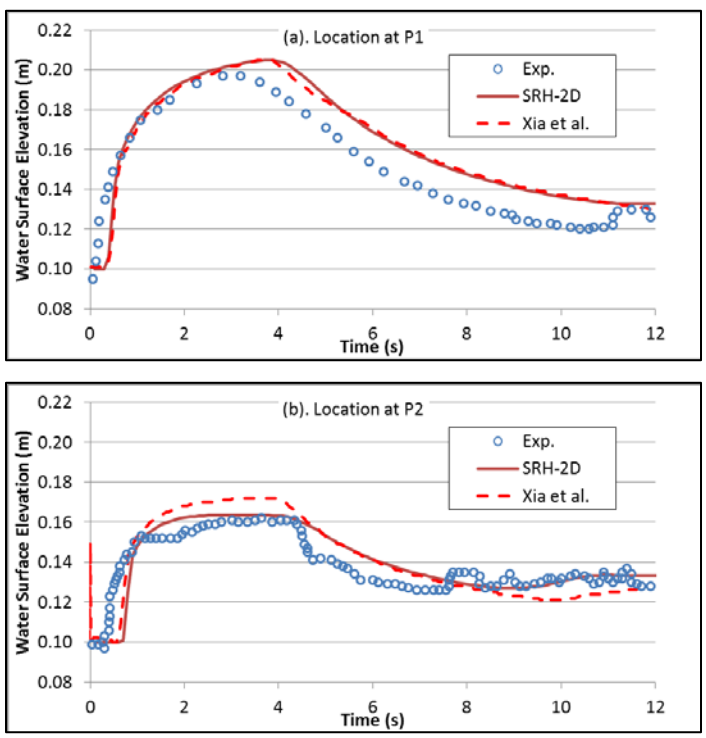

Figure 3 Comparisons between the observed, Xia et al. (2010) calculated, and current SRH-2D calculated water surface elevations

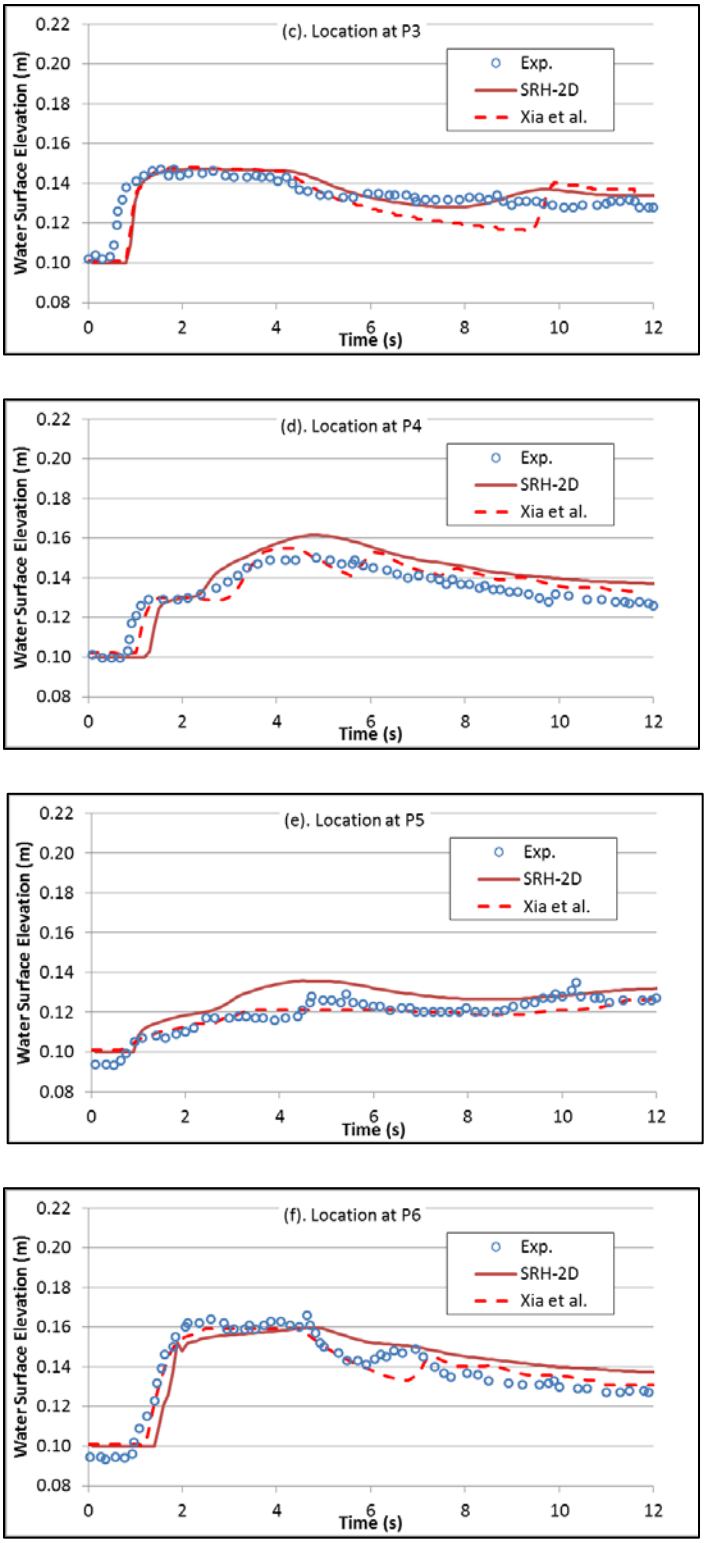

Figure 3 Continued

Figure 4 shows the comparisons of the bed profiles observed in laboratory, simulated by Xia et al. (2010), and simulated by SRH-2D. The SRH-2D simulation includes the discussed extra terms related to hyperconcentrated

flow. The sediment erosion and deposition were simulated using the sediment transport capacity functions of Wu and Long (1993) for suspended load and Dou et al. (1999) for bed load. At CS1 (x=4.1 m), the maximum bed scouring depth was predicted well, however, the model under-predicted the deposition on the left side of the channel after expansion. At CS2 (x=4.4), the model over-predicted the channel erosion and under-predicted channel deposition. A circulation zone is created by the channel expansion, and the deposition rate is closely related to the size of the circulation zone and the turbulence within it.

The test case was also simulated using the erosion and deposition equations for hyperconcentrated flow of Cao et al. (2004 and 2006, and Eqs. and) and the sediment transport capacity function of Engelund and Hansen (1966). Cao et al.'s equations over-predicted channel erosion by one order of magnitude and Engelund and Hansen's equation under-predicted the erosion by one order of magnitude. Both of these equations are intended for rivers, but the Engelund and Hansen formula was not developed for hyperconcentrated flows.

The following conclusion can be drawn from this testcase:

- The numerical model predicts the water surface elevation with high accuracy in a suddenly expanded channel.
- The numerical model reproduces the channel erosion well, however with less accuracy compared with the simulation of water surface elevation. The numerical model under-predicts the sediment deposition in the circulation zone.
- The sediment erosion and deposition rate can be calculated directly from corresponding equations or indirectly from sediment transport capacities. The correct simulation of the channel bed depends on the correct selection of these equations and model calibration.

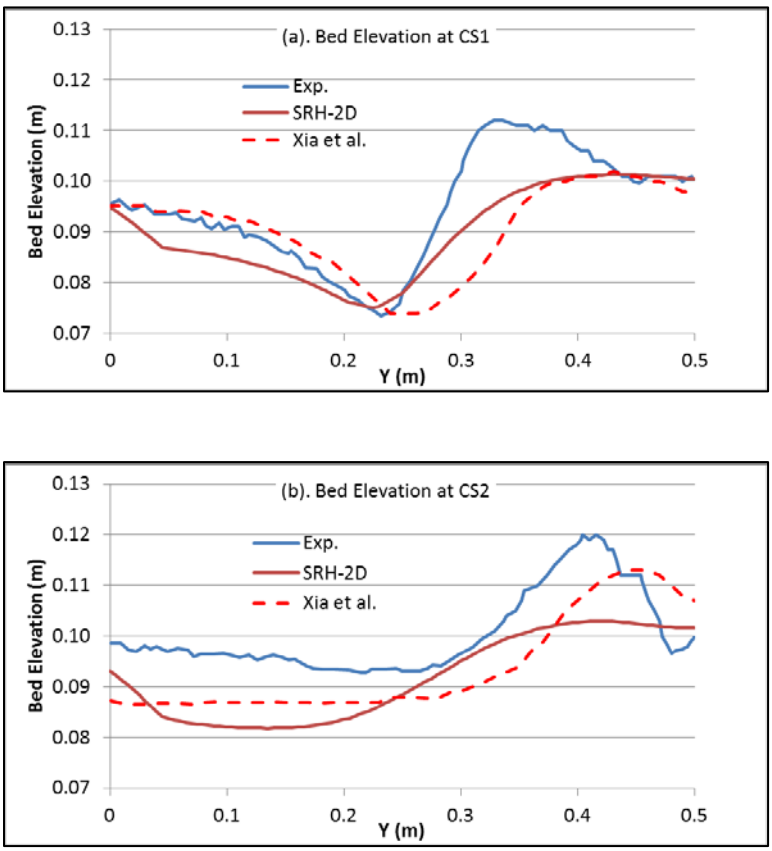

Figure 4 Comparisons between the observed, Xia et al. (2010) calculated, and current SRH-2D calculated water surface elevations

Summary

The current SRH-2D model developed at Bureau of Reclamation is extended to include the effect of sediment concentration on sediment fall velocity, sediment erosion and deposition, density variance in the momentum equation, and the exchange of flow and bed in the mass and momentum equations. The numerical model is tested and validated using two cases : an idealized transient flow and sediment process due to a dam-break flow over a horizontal bed and a more complex dam-break flow with a sudden enlargement. It is shown that the modifications are necessary for hyper-concentrated flow simulation. Also, the new model is validated and is found to predict the observed water surface elevation fairly well.

The study also finds that hyperconcentrated sediment-laden flow alters the flow density and the particle fall velocity. Currently equations for sediment erosion and deposition rates for a hyperconcentrated flow are based on laboratory data or are site specific. They should be examined before they can be applied to actual field conditions.

The extra terms related to the density gradient and the terms related to the exchange between the flow and the erodible bed in hyperconcentrated flow increase the water front speed and wave height during a dam break.

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