Abstract: The empirical dataset of surveyed sand bar volumes in Marble Canyon on the Colorado River downstream of Glen Canyon Dam is analyzed. A subset of the empirical dataset is established based on bar and survey date consistency, resulting in the calibration dataset. The sand bar calibration dataset is represented by both the mean and median of the sand bar volume.

A conceptual model describing erosive and depositional processes for sand bars in Marble Canyon is described. A series of model formulations (termed Model V0 through V5) are developed and Model V3 is selected based on model performance calculated as the normalized sum of squared errors and adjusted R-squared. Confidence intervals are developed for the parameters and for the predicted bar volumes coinciding with calibration data survey dates.

Median and mean versions of Model V3 show similar percent improvement over the model V0 performance (86% for median, 89% for mean). The prediction confidence intervals contain 19 of the 28 observed bar volumes for both the mean and median V3 models. Deposition and erosion trends between survey dates are correctly predicted 25 out of 28 times for both the mean and median V3 models. The mean V3 model tended to perform better in the vicinity of high flow experiments (HFEs) as compared to the median V3 model.

INTRODUCTION

The Department of the Interior, through the Bureau of Reclamation (Reclamation) and the National Park Service (NPS) are preparing an environmental impact statement (EIS) for the adoption of a long-term experimental and management plan (LTEMP) for the operation of Glen Canyon Dam on the Colorado River. The EIS will fully evaluate dam operations and identify management actions and experimental options that will provide a framework for adaptively managing Glen Canyon Dam over the next 15 to 20 years.

The Sand Bar Volume Model (SBVM) was developed by Reclamation’s Technical Service Center specifically for performing alternative analysis during the LTEMP EIS process. The sediment resource goal for the LTEMP EIS is “to increase and retain fine sediment volume, area, and distribution in the Glen, Marble and Grand Canyon reaches above the elevation of the average base flow for ecological, cultural, and recreational purposes." The Sand Budget Model (Wright et al., 2010) is useful for quantifying the overall sand budget within a reach, but does not consider the proportion of that sediment in the bed versus bars, and therefore does not explicitly reflect the sediment resource goal. The intent of SBVM is to represent a time series of all the bars in Marble Canyon by incorporating the empirical sand bar data, which is described in terms of sediment volumes relative to different reference elevations, into an analysis that more directly reflects the sediment resource goal. SBVM is only applicable to the Colorado River downstream of Glen Canyon Dam, and has currently been calibrated to the dataset consisting of sand bars in Marble Canyon (Figure 1).

REVIEW OF EMPIRICAL DATA

The U.S. Geological Survey’s Grand Canyon Monitoring and Research Center (GCMRRC) in collaboration with Northern Arizona University (NAU) have been performing repeat surveys of select bars throughout Marble and Grand Canyon for decades (Mueller et al., 2014). There are 25 bars in Marble Canyon, eight in Upper Marble Canyon (RM 0 to RM 30) and 17 in Lower Marble Canyon (RM 30 to RM 61) with 42 survey dates reported between 9/15/1990 and 9/21/2013. Data collected quantify the sediment area and volume relative to the water surface elevations (WSE) associated with a flow of 8,000 cubic feet per second (ft³/s) and 25,000 ft³/s (Grams, 2013). Data were binned as volume/area less than the 8,000 ft³/s WSE, between the 8,000 and 25,000 ft³/s WSE, and greater than the 25,000 ft³/s WSE. To reflect the sediment resource goal of the LTEMP EIS, the sand bar data above the 8,000 ft³/s WSE (8,000-25,000 ft³/s and greater than 25,000 ft³/s) is considered to represent the sediment above the elevation of the average base flow. The sediment below the 8,000 ft³/s WSE is addressed with the Sand
Budget Model (Wright et al., 2010); the SBVM focuses on the portion of sediment referenced in the sediment resource goal of the LTEMP EIS.

Figure 1 Location map: primary area of interest is Marble Canyon (Lees Ferry to Little Colorado River)

A consistent set of sand bars and survey dates was developed to be used as the calibration dataset. The development of the calibration dataset attempted to maximize the time span of observations but also maintain a consistent set of the highest number of sand bars possible. This approach yielded a calibration dataset of 29 dates (out of 42) and 14 sand bars (out of 25) from 7/26/1991 to 9/21/2013.

The SBVM represents a time series of all the bars in Marble Canyon, where the conglomeration of bars is represented by a single value; either the mean or median volume of those bars. Figure 2 is the distribution of bar volumes above the 8,000 ft³/s WSE for a reported survey date. Note that the mean is always greater than the median, and that there is more variability in the median through time as compared to variability in the mean. The minimum and maximum bar volume is also shown in the graph; the maximum is always associated with 51-mile bar, the minimum is either 3-mile or 16-mile bar. An overarching assumption of the SBVM is that GCMRC selected bars to survey because those bars represent the variability in morphology and type found in the canyons, and that any inferences made via calibration to the surveyed bars would be representative of all of the sand bars, surveyed or not.

The surveys immediately after HFE events (1996, 2004, and 2008) all have mean volumes greater than 290,000 cubic feet (ft³). Note that the HFE signal is not as pronounced in the median record; the surveys post-2004 and 2008 HFEs register a median value less than the survey that is approximately five months after the 1996 HFE. The time series of mean bar data likely represents the average conditions in the reach more accurately than the median, but both time series will be used as calibration datasets during model selection.

The trends in the mean and median data generally are the same except for the period from the late 1990s to the early 2000s. Figure 3 presents the correlation between the mean and median bar volume of the calibration dataset.
The impetus of model development was to find a way to predict bar responses to High Flow Experiments (HFE), sometimes referred to as controlled floods. The controlled high flow releases are meant to mimic natural flooding to an extent and benefit sediment-dependent resources such as sandbars. Building on the body of knowledge of sandbar behavior in Glen, Marble, and Grand Canyon – largely developed by GCMRC (Wiele et al., 2007; Wright and Kaplinski, 2011) – a general conceptual model was developed. The two fundamental processes which need to be captured are bar building (deposition) and bar erosion.

Sand bar deposition and bar building occur at high discharge and with high sediment concentration. The rate of bar building is largest during the early stages of an HFE, and tapers off as the bar becomes more ‘full’; there is a physically reasonable maximum bar volume for each bar (Wiele and Torizzo, 2005). Also, the more ‘empty’ a bar
is, the higher the potential volume of deposition. An important aspect of deposition is the potential volume of sediment which a bar may contain.

The rate of sand bar erosion tends to be large during the first few to six months after a significant bar building event, then tapers off to a fairly constant rate. Early SBVM formulation experimented with combining depositional and erosional processes into a single term to capture bar volume change, but it soon became apparent that these two processes are governed by different processes under different conditions. The following discusses the model as consisting of two terms: one for depositional processes and one for erosional processes.

The general model formulation for a tool that can be used to predict bar volumes is presented in equation 1. There is a deposition rate and an erosion rate calculated at each time step in the model. The equation can be time-integrated get equation (2). A change in bar volume for a given time step is calculated by taking the difference between the erosion and deposition rates and converting to a volume by multiplying by time step and a porosity term. The bar volume for the next time step is the sum of the current time steps bar volume and the change in bar volume calculated at that time step (equation 3).

\[
\frac{dS_b(1-\eta)}{dt} = Q_d - Q_e
\]  

(1)

where:

- \( S_b \) = mean sand bar volume (ft\(^3\)) above a base flow rate water surface elevation
- \( \eta \) = porosity of sediment (-)
- \( Q_d \) = depositional flux of sediment into sand bar (ft\(^3\)/s)
- \( Q_e \) = erosional flux of sediment from sand bars (ft\(^3\)/s)
- \( t \) = time (s)

\[
\Delta S_b(1-\eta) = \int_{t_i}^{t_f} (Q_d - Q_e) dt
\]  

(2)

\[
S_{b(i+1)} = S_{b(i)} + \Delta S_{b(i)}
\]  

(3)

Five different sets of depositional and erosional terms are presented below. These five sets represent different models of depositional and erosional behavior; the erosion term \( Q_e \) in models V3 – V5 are identical and are paired with different depositional equations. Models V1-V5 can be compared to the average bar volume, identified as V0. That is, V0 assumes that the predicted bar volume for any time step is simply the average (mean or median as appropriate) bar volume. The model error associated with V0 will be a benchmark to which the model performance for V1-V5 can be compared.

**Model V1: Flow only**

\[
Q_d = a_d Q_x^{b_d}
\]

\[
Q_e = a_e Q_x^{b_e}
\]

**Model V2: Storage only (\( Q \) is used indirectly to calculate \( S_{bv} \), see below)**

\[
Q_d = a_d \left( \frac{S_{bv}}{S_b} \right)^m
\]

\[
Q_e = a_e \left( \frac{S_b}{S_{bv,max}} \right)^n
\]

**Model V3: Flow and Storage (no concentration term)**

\[
Q_d = a_d Q_x^{b_d} \left( \frac{S_{bv}}{S_b} \right)^m
\]

\[
Q_e = a_e Q_x^{b_e} \left( \frac{S_b}{S_{bv,max}} \right)^n
\]

**Model V4: Flow, Storage, Concentration (exponent on \( Q \))**

\[
Q_d = a_d C Q_x^{b_d} \left( \frac{S_{bv}}{S_b} \right)^m
\]
\[ Q_e = a_e Q_x^{b_e} \left( \frac{S_B}{S_{bv,\text{max}}} \right)^n \]

Model V5: Flow, Storage, Concentration (exponent on product of QC)
\[ Q_d = a_d (C Q_x)^{b_d} \left( \frac{S_{bv}}{S_B} \right)^m \]
\[ Q_e = a_e Q_x^{b_e} \left( \frac{S_B}{S_{bv,\text{max}}} \right)^n \]

Where:
- \( Q_d \) = deposition rate at time step \( i \) (ft\(^3\)/s)
- \( Q_e \) = erosion rate at the time step \( i \) (ft\(^3\)/s)
- \( Q_x \) = volumetric water discharge in Colorado River (ft\(^3\)/s) at River Mile 30 divided by 2,000 ft\(^3\)/s (-);
- \( C \) = volumetric sand-sized sediment concentration within the Colorado River at River Mile 30 (-)
- \( a_d, b_d, m \) = calibration parameters (ft\(^3\)/s), (-), (-), respectively
- \( a_e, b_e, n \) = calibration parameters (ft\(^3\)/s), (-), (-), respectively.
- \( S_{bv} \) = maximum available sand bar volume below the WSE at specific flow rate, \( Q \) (ft\(^3\)).
- \( S_{bv,\text{max}} \) = available sand bar volume at maximum flow rate of 45,000 ft\(^3\)/s (ft\(^3\)).

The calibration parameters are collectively called a calibration parameter set and include \( a_d, b_d, m, a_e, b_e, \) and \( n \).

The variable \( S_{bv} \) is derived from the empirical volume data for all available survey dates for the 14 bars used in the calibration dataset. \( S_{bv} \) is a variable describing the maximum potential sand bar volume at different flow rates. We currently have three data points to which to fit a curve and these points were developed as such:

- A zero sand bar volume was assigned to a discharge of 8,000 ft\(^3\)/s, as we are only concerned with the volume of sand above the 8,000 ft\(^3\)/s WSE.
- The maximum volume for each of the 14 bars associated as being between the 8,000-25,000 ft\(^3\)/s WSE were averaged (mean and median) and assigned to being the potential bar volume at 25,000 ft\(^3\)/s.
- The volume between the 8,000-25,000 ft\(^3\)/s WSE and the volume above the 25,000 ft\(^3\)/s WSE were summed for each survey date, and the maximum of those sums for each of the 14 bars were averaged (mean and median) and assigned to being the potential bar volume at 45,000 ft\(^3\)/s, which is the maximum planned release flow rate under the interim guidelines barring an excessively wet hydrologic year.

It is assumed for the sake of the \( S_{bv} \) equation that the available sand bar volume at a particular flow rate does not change in time. Further, a continuous function for the \( S_{bv} \) equation was desirable for optimization purposes. A logistic function was found to fit the data well and be continuous, once the discharge was scaled. Dividing the discharge by 2,000 ft\(^3\)/s provided the logistic functions for mean and median data (Figure 4).

As described in the presentation of the deposition and erosion models, the flow terms use the scaled discharge \( Q_x \) (\( Q/2000 \)) and not the discharge \( Q \), based on general early observations that a smaller base upon which an exponent is placed improves model performance. Also, dividing the flow by 2,000 ft\(^3\)/s is consistent with the flow terms in the model.

All models were implemented into the Mathworks® Matlab software. The modified Sand Budget Model (Wright et al., 2010; Russell and Huang, 2010) was developed as a sediment budget for Marble and Eastern Grand Canyon. Because we are focusing on the bars in Marble Canyon, the discharge and concentration time series at the middle of the canyon (RM30) is used as input to SBVM. The initial condition 7/26/1991 for the SBVM model is the average bar volume of 269,809 ft\(^3\) for the mean and 247,433 ft\(^3\) for the median of the calibration data. Because there are multiple parameters and the model is non-linear, optimization is dependent on the initial guess of parameter values. To increase the confidence in the results, a brute force grid search was employed for all models to locate the region of the global maximum, not just a local maximum. Parameter search domains for the mean and median model simulations were: 1E-20 to 1E20 for \( a_d \) and \( a_e \); 0 to 30 for \( m \) and \( n \); and at least -8 to 6 for \( b_d \) and \( b_e \), although the search domain for \( b_d \) and \( b_e \) tended to be extended from -12 to 12 because the solution domain showed good performance near the bounds of the initial search domain.
MODEL SEARCH DOMAINS

The bounded search optimization routine “Fminsearch” in Matlab was utilized once the best performing parameter sets were identified from the grid search. Multiple initial guess parameter sets were optimized for each model based on the results of the grid search.

OPTIMIZATION AND MODEL PERFORMANCE

**Model V0: Average of calibration dataset** This is the simplest model and is used as a benchmark to assess the results of models V1-V5. In this model, the predicted bar volumes are constant through time and are the mean for the mean bar volumes, and the median for the median bar volumes. Model performance is assessed as the sum of squared error (SSE) normalized to the appropriate average bar volume. The SSE for a mean bar volume of 267,754 ft$^3$ (normalized to this mean) is 0.125, and for a median bar volume of 204,013 ft$^3$ (normalized to this median) is 0.958. Figure 5 depicts the observed and the model V0 predicted bar volume above the 8,000 ft$^3$/s WSE.
**Model V1: Flow Only** This model uses only a flow term to predict erosion and deposition. There are no history effects accounted for in the erosion and deposition rates, meaning the bar volume existing at a given time step does not inform the rates of erosion and deposition. There is also no utilization of the $S_{bv}$ curve relating the flow rate in the river to the presumed potential bar volume associated with that flow rate.

For both the mean and median cases, the top performing parameter sets (combinations of $b_d$, $b_e$, $a_d$, $a_e$) resulting from the grid search were used as initial guesses for the bounded Matlab optimization routine “Fminsearch”, with the bounds coinciding with the step size specified during the grid search. For example, the grid search for this model had a step size of two for the exponents ($b_d$, $b_e$) and for the coefficients ($a_d$, $a_e$) the step size was two orders of magnitude. During the optimization for each parameter set, the search bounds were set to ±2 on the exponents and ±2 orders of magnitude on the coefficients.

The parameter sets resulting from the bounded optimization were plotted against their performance (Figure 6). The median optimization plot shows the behavior one would expect; the range for a given parameter decreases as the SSE decreases (performance increases). This behavior appeared for the V1 mean model as well, and gives greater confidence in the optimal parameter set than if the opposite were true (larger range for a given parameter as performance increased).

![Figure 6 Parameter set performance after optimization for median V1 model](image)

**Model V2: Storage Only** This model uses only a storage term to predict erosion and deposition. The flow rate at a given time step does not inform the rates of erosion and deposition, except for indirectly, where the flow rate selects the potential bar volume, $S_{bv}$, and thus the deposition is indirectly associated with flow rate. The erosion rate is simply calculated as a ratio of the current time step bar volume relative to the maximum bar volume; the smaller the bar gets, the slower the rate of erosion.

For the median case, the top performing parameter sets (combinations of $m$, $n$, $a_d$, and $a_e$) were used as initial guesses for the Matlab optimization routine “Fminsearch” with the bounds coinciding with the step size specified during the grid search as described for model V1. Due to the smooth nature of the mean performance surface, only one optimization was run.

The parameter sets resulting from the bounded optimization for the median data were plotted against their performance. The median optimization plot showed a different pattern than the median optimization plot for model V1; namely, there does not appear to be a convergent solution as the performance increases. Less confidence should be given to this model due to the non-convergent nature of the parameter sets relative to performance. The grid
search for Model V2 mean produced a smooth solution surface that leads to greater confidence in the optimized solution.

**Model V3: Flow and Storage** This model uses a flow term and a storage term to predict both erosion and deposition. This model is a combination of models V1 and V2. Flow rate and current bar volume inform the rates of erosion and deposition for that time step.

Only those parameter sets from the grid search for the mean model that resulted in an SSE less than or equal to one-half the SSE from model V0 were carried forward to optimization (n=429). If the same selection criteria was applied to the results of the grid search for the median model (using those that had an SSE equal to one-half the model V0 SSE), only about 1% of the parameter sets for the median model would have moved forward to optimization. To increase the sample set, the slope of a Weibull-distribution cumulative distribution function (CDF) was investigated. In the vicinity of an SSE = 0.577 the slope of the CDF transitions from variable to relatively constant. This location corresponds to 6% on the CDF, so the top 6% performing parameters sets from the median grid search (n=1226) moved forward to optimization.

The top performing parameter sets were used as initial guesses for the Matlab optimization routine “Fminsearch” with the bounds coinciding with the step size specified during the grid search as described for models V1. The parameter sets resulting from the bounded optimization were plotted against their performance (Figure 7). The optimization plot shows a pattern that would be expected in optimization; each parameter appears to converge as the performance increases (SSE decreases). This behavior suggests more confidence in the model and the optimized parameter set.

**Model V4: Flow, Storage, and Concentration (linear)** This model uses a flow term and a storage term to predict both erosion and deposition, much like Model V3, except that the addition of a concentration term is included to predict deposition rates. The exponent on the concentration term is fixed at 1.

Only those parameter sets from the grid search for the mean model that resulted in an SSE less than or equal to one-half the SSE from model V0 were carried forward to optimization (n=1410). If the same selection criteria was applied to the results of the grid search for the median model, (using those that had an SSE equal to one-half the model V0 SSE), only about 3% of the parameter sets for the median model would have moved forward to optimization. To increase the sample set, the slope of the CDF was investigated. In the vicinity of an SSE = 0.517 the slope of the CDF transitions from variable to relatively constant. This location corresponds to 5% on the CDF, so the top 5% performing parameters sets from the median grid search (n=1021) moved forward to optimization.

The top performing parameter sets were used as initial guesses for the Matlab optimization routine “Fminsearch” with the bounds coinciding with the step size specified during the grid search, as discussed in model V1. The parameter sets resulting from the bounded optimization for the median data are plotted against their performance.
similar to Figure 6 and Figure 7. The median and mean optimization plot shows a similar convergence of the erosion parameters ($n$, $b$, and $a$) as for model V3. However, the deposition parameters ($m$, $b$, and $a$) do not appear to be converging with improved performance. In addition, the performance for model V4 does not achieve the same level as model V3, whether mean or median. This behavior not only reduces the confidence in the optimized parameter sets but also reduces the confidence in the model. The following model (V5) will allow the exponent on the concentration parameter to vary in an attempt to improve performance.

**Model V5: Flow, Storage, and Concentration (power)** This model uses a flow term and a storage term to predict both erosion and deposition, with the addition of a concentration term in the prediction of deposition rates. Unlike model V4, the concentration term has an exponent that is allowed to vary. The exponent is defined to equal the exponent on the flow term ($b$) so that the number of parameters does not increase.

Only those parameter sets from the grid search for the mean model that resulted in an SSE less than or equal to $\frac{1}{2}$ the SSE from model V0 were carried forward to optimization ($n=380$). If the same selection criteria was applied to the results of the grid search for the median model, (using those that had an SSE equal to $\frac{1}{2}$ the model V0 SSE), only about 1.3% of the parameter sets for the median model would have moved forward to optimization. To increase the sample set, the slope of the CDF was investigated. In the vicinity of an SSE = 0.617 the slope of the CDF has a noticeable break. This location corresponds to 6% on the CDF, so the top 6% performing parameters sets from the median grid search ($n=1226$) moved forward to optimization.

The top performing parameter sets were used as initial guesses for the Matlab optimization routine “Fminsearch” with the bounds coinciding with the step size specified during the grid search as described for models V1. The parameter sets resulting from the bounded optimization are plotted against their performance and the median and mean optimization plot showed convergence of the erosion and deposition parameters. The performance for model V5 does not achieve the same level as model V4, whether mean or median. This behavior reduces the confidence in the model.

**Summary of Optimized Models** Figure 8 presents a summary of the model performance for the mean and median dataset as a progression through models V0-V5. The percent improvement of the models compare models V1-V5 relative to the initial V0 model.

In both the mean and median cases, model performance improves from V0 through V3. Adding the concentration term reduces model performance. It is plausible that the explanation for this lies with the fact that the concentration used is a time series output from the modified Sand Budget Model rather than the actual measured data. Figure 6 from the Wright et al. (2010) paper showing that a significant amount of scatter exists in a concentration vs. discharge plot (not atypical) at River Mile 30. It is possible that using the measured (not model) concentration would improve model performance when including the concentration time series. However, the approach of using measured data means the model is no longer predictive under future flow and operational scenarios.

![Figure 8](image-url)
It is apparent that model V3 is the best for both the mean and median datasets. It is the first model in the progression from V1-V5 that has six regressors, (models V1 and V2 had 4 each). Calculating an adjusted $R^2$ (or $\hat{R}^2$, r-bar squared) can help assess whether the added complexity is justifiable (http://www.mathworks.com/help/curvefit/evaluating-goodness-of-fit.html). Equation 4 presents the equation used to calculate the adjusted $R^2$ and Table 1 presents the results by model.

$$R^2 = 1 - \frac{SSE(n-1)}{SST(v)}$$  \hspace{1cm} (4)

Where:

- $SSE$ = sum of squared error (or sum of squared residuals)
- $SST$ = total sum of squares (= regression sum of squares + residual sum of squares)
- $n$ = number of response values
- $v$ = residual degrees of freedom = $n-m$
- $m$ = number of fitted coefficients

Table 1 Adjusted R-squared for models V1-V5; mean and median

<table>
<thead>
<tr>
<th>Model</th>
<th>V1</th>
<th>V2</th>
<th>V3</th>
<th>V4</th>
<th>V5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
<td>28</td>
<td>28</td>
<td>28</td>
<td>28</td>
<td>28</td>
</tr>
<tr>
<td>$m$</td>
<td>4</td>
<td>4</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>$v$</td>
<td>24</td>
<td>24</td>
<td>22</td>
<td>22</td>
<td>22</td>
</tr>
<tr>
<td>$R^2$, median</td>
<td>0.27</td>
<td>0.66</td>
<td>0.82</td>
<td>0.74</td>
<td>0.74</td>
</tr>
<tr>
<td>$R^2$, mean</td>
<td>0.16</td>
<td>0.55</td>
<td>0.87</td>
<td>0.81</td>
<td>0.81</td>
</tr>
</tbody>
</table>

The adjusted R-square values justify the additional complexity of transitioning from model V2 (with 4 regressors) to V3 (with 6 regressors), and suggest that models V4 or V5 do not offer improvement over V3.

CONFIDENCE INTERVALS

The Mathworks Matlab functions “nlparci” and “nlpredci” were used to develop the parameter confidence intervals and the prediction confidence intervals, respectively, both at the 95% confidence level. Table 2 presents the parameter confidence intervals for the median and mean datasets for model V3. Figure 9 presents the predicted sand bar volumes along with the predicted confidence intervals at the time steps corresponding to the observations.

Table 2 Parameter confidence intervals (CI) for model V3

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Lower CI Optimized</th>
<th>Upper CI</th>
<th>Lower CI Optimized</th>
<th>Upper CI</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_d$</td>
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<td>4.97E-08</td>
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</tr>
<tr>
<td>$a_e$</td>
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<td>4.83E-05</td>
<td>2.09E-04</td>
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</tr>
<tr>
<td>$b_d$</td>
<td>3.593</td>
<td>5.015</td>
<td>6.437</td>
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</tr>
<tr>
<td>$b_e$</td>
<td>0.924</td>
<td>2.272</td>
<td>3.620</td>
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</tr>
<tr>
<td>$m$</td>
<td>0.993</td>
<td>4.697</td>
<td>8.400</td>
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<td>$n$</td>
<td>7.105</td>
<td>11.664</td>
<td>16.224</td>
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</tr>
</tbody>
</table>

Both the mean and median prediction confidence intervals contain 19 of the 28 observed bar volumes (the first of the 29 observations – 7/26/1991 – was used as an initial condition for the model). The notable errors of the mean model are that it generally does not capture the cluster of measurements in summer/fall 1997, it under-predicts the erosion after the small depositional event in Fall 2000, and under-predicts the small depositional event in Fall 2007.
The notable errors of the median model are that it over-predicts the erosion before the 1996 HFE, under-predicts the deposition during the 2004 HFE, and under-predicts the erosion after the 2008 HFE.

Figure 9 Model V3 predicted bar volume time series and prediction confidence intervals (median above, mean below)
Comparing observed vs. predicted trends of deposition or erosion between survey dates show that both models predict the trend correctly 25 out of 28 times. Both models miss the apparently mild depositional event between October 2006 and October 2007. The remaining trend errors for both the mean and median models occur in the 2000 to 2003 time frame.

Some parameter confidence intervals bound zero. The parameter confidence intervals developed by Matlab assume a normal parameter distribution, and this is likely not the case for the nonlinear model. Also, regression typically assumes that parameters are not correlated. An investigation of the optimized parameters shows that there is a correlation between the coefficients ($a_d$, $a_e$) and the flow exponents ($b_d$, $b_e$) respectively, as well as correlation between the coefficient $a_e$ and the storage exponent $n$. No apparent correlation exists between the coefficient $a_d$ and the storage exponent $m$. For physically practical purposes, a lower bound of zero should be imposed on all confidence intervals in Table 2.

**SUMMARY**

The empirical dataset of surveyed sand bar volumes in Marble Canyon on the Colorado River downstream of Glen Canyon Dam was analyzed. A subset of the empirical dataset was established based on bar and survey date consistency, resulting in the calibration dataset. The sand bar calibration dataset is represented by both the mean and median of the sand bar volume.

A conceptual model describing erosive and depositional processes for sand bars in Marble Canyon is described. A series of model formulations (termed Model V0 through V5) were developed and Model V3 is selected based on model performance calculated as the normalized sum of squared errors and adjusted R-squared. Confidence intervals were developed for the parameters and for the predicted bar volumes coinciding with calibration data survey dates.

 Median and mean versions of Model V3 show similar percent improvement over the model V0 performance (86% for median, 89% for mean). The prediction confidence intervals contain 19 of the 28 observed bar volumes for both the mean and median V3 models. Deposition and erosion trends between survey dates are correctly predicted 25 out of 28 times for both the mean and median V3 models. The mean V3 model tended to perform better in the vicinity of HFEs as compared to the median V3 model.

**REFERENCES**


