# FLOW ENERGY AND BEDLOAD-TRANSPORT EFFICIENCY: THE FROUDE NUMBER AS A METRIC FOR BEDLOAD TRANSPORT RATES

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## INTRODUCTION

The study reported in this short paper is preliminary in nature. The ideas and impetus behind this work stem from decades of research on unstable channels and was initiated in the 1990's as part of studies on channel evolution of the Toutle River System in the aftermath of the 1980 eruptions of Mount St Helens (Simon, 1992; 1999). This earlier work included a strong focus on trends of non-linear decay in flow energy and the processes (Graf, 1977) responsible for minimizing energy in an adjusting alluvial stream (Simon and Thorne, 1996). Applied to the suite of extremal hypotheses published in the 1970's and 1980's and placed within the framework of open systems theory with its references to minimizing entropy in landscapes, the associated theory that conditions at critical flow represent the most efficient means for channels to transmit water (Blench, 1966) and sediment (Kirkby, 1977) is considered. If this is to be realized, then maximum sediment transport should occur at hydraulic conditions representing minimum energy for that discharge (ie. critical flow; Froude number equals 1.0). To test this hypothesis, unit bedload transport data and associated hydraulic data were required, which span subcritical and super-critical flow regimes.

Geomorphically we can describe these adjustments in terms of measureable changes in geometry, its effects on boundary roughness and associated hydraulic conditions. Channel widening, particularly when associated with degradation, was found to be the most effective process in reducing flow energy, as all three components of total mechanical energy are reduced (ie. datum [elevation], hydraulic depth, and velocity) (Simon, 1992; Simon and Thorne, 1996). Further, energy slope represents the dissipation of energy over the reach. At Mount St. Helens with its braided channels, plentiful sediment supply, and cohesionless banks and terrace slopes, channel widening is and will remain (assuming no additional disturbances), the dominant process on the North Fork Toutle River for the next century (Simon and Klimetz, 2012). Channel widening of hundreds of meters has resulted in extremely shallow flow depths and frequent observations of critical-flow conditions (standing waves). Similar observations are common on braided streams. Over the course of adjustment, wide, shallow channels form where some peak flows oscillate between sub-critical and super-critical flow regimes (Simon, 1992) trending towards critical. The results further allow for an energy-based explanation for the hierarchy of channel-adjustment processes, their role in minimizing flow energy, and the importance of both bed and bank erosion in channel evolution (Simon and Thorne, 1996).

Sediment transport in rivers and streams is typically predicted as a function of shear-stress based theory, which when applied to bulk properties of channelized streamflow, places a large importance on the depth of flow and water surface (or energy) slope. This theory is based on the Law of the Wall, a logarithmic velocity gradient with increasing velocity with distance above the bed, with a corresponding decrease in boundary shear stress. It is these conditions that are responsible for rotating sediment grains out of pockets on the bed surface. Accordingly, a larger flow depth for a given slope produces a higher integral of shear stress applied to a bed particle and, therefore, a higher sediment flux rate. Such theory and its inclusion in sediment transport equations may not accurately represent observed sediment transport rates in a range of natural environments including sandy beach shorelines, alluvial fans, river bars, and floodplains, where flow is characteristically shallow, bed slopes are steep (≥1%), and beds are generally smooth with larger particles sparsely accumulated over a finer substrate. In such environments, vertical velocity and shear stress gradients may not be relevant at the scale of particles on the bed surface and sediment may be entrained in flows that are near the length scale of particles on the bed.

## NEED FOR NEW SEDIMENT-TRANSPORT THEORY

Shear stress based theory tends to predict zero transport under such conditions because the critical flow depth has not been reached, yet particles in these environments typically move at high rates over smooth beds (Laronne and Reid, 1993) in rapid, shallow flows approaching or exceeding the critical value (Froude number ≥ 1):

$$F_r = \frac{u}{\sqrt{gh}} \tag{1}$$

Where $u$ = downstream flow velocity, in m/s; $g$ = acceleration of gravity, in m/s$^2$; and $h$ = flow depth in m.

We hypothesize that in such rapid, shallow flows the processes by which sediment moves are not well characterized by theory designed for particle rotation out of pockets. Rather, they are more reasonably responsive to a square wave of flow pushing along the stoss particle boundary. In these shallow, rapid flow regimes, sediment particles on the bed are typically not encumbered by clast support, so transport only requires a flow force large enough to push particles with median diameters far greater than the bed roughness scale (e.g., mean friction angle (Buffington et al., 1992; Kirchner et al., 1990)), and do not rotate out of pockets. Once entrained, these relatively coarse particles may continue to move as long as this flow force is maintained and the bed surface remains smooth (preventing grain to grain interactions that would slow them to a halt).

Shear stress approaches are typically very sensitive to empirically defined values of critical shear stress for entrainment, which are known to vary dramatically for non-uniform sediment mixtures (Wiberg and Smith, 1987; Buffington and Montgomery, 1997). Based on the understanding that entrainment is sensitive to bed material grain-size distribution, critical shear stress is typically

computed as a power law of the ratio of particle sizes subject to movement to median bed-grain sizes (Andrews, 1983; Egiazaroff, 1965) or more recently, by sand fraction in the bed (Wilcock and Crowe, 2003), both of which are assumed to account for the effects of relative grain protrusion/hiding and pocket angle rotation on entrainment.

In cases of rapid shallow flow over smooth beds, these calculations of critical (or reference) shear stress yield values near zero, suggesting that any positive value of flow will move sediment particles on the bed surface (i.e., there are no frictional forces to overcome associated with clast support). Even assuming near zero critical shear stress, surface based equations that were designed for gravel-bed rivers (e.g., (Parker, 1990; Wilcock and Crowe, 2003)) still may not perform well under shallow rapid flow regimes over smooth, unarmored beds lacking developed bar morphology (Hassan, 2005; Laronne et al., 1994; Singer and Michaelides, 2014). Consequently, re-calibration of these equations has been undertaken to overcome the limitations in these approaches and to more accurately hindcast high observed flux rates (e.g., ephemeral dryland sediment flux in Nahal Eshtemoa, Israel (Powell et al., 2003)). However, these empirical tweaks to existing equations do not yield new insight into transport processes within shallow, rapid flow regimes. Thus, we in the sediment transport community continue to grapple with the process controls on flux rates and morphologic development in many such fluvial environments. Here, we re-investigate published sediment flux rates for controlled conditions within a range of field and flume datasets and the corresponding hydraulics to develop new insight into sediment transport under rapid, shallow flow regimes.

## FLOW-ENERGY BASED HYPOTHESIS

Controversial theories of minimum stream power and rate of energy dissipation were developed in the late 1970's and early 1980's as a means of describing the equilibrium shape of alluvial channels (Yang, 1976; Chang, 1979; Yang and Song, 1979; Chang, 1980; Yang et al., 1981). These theories were the subject of extensive debate in the geomorphic and engineering literature. Chorley (1962) describes channel adjustment in terms of open systems theory "*...wherein the import and export of energy and material are equated by means of an adjustment of the form, or geometry, of the system itself*." (Chorley, 1962, p. B3). Simon (1992) reported that the theories were applicable to the adjustment of alluvial streams, noting that channel widening and reduction in flow depths was a critically important process in minimizing flow energy. This work further showed that equilibrium geometries for the North Fork Toutle River tended towards a minimum specific energy, representing a maximum unit discharge, critical flow and a Froude number of 1.0 (Simon and Thorne, 1996). Blench (1966) and Ergenzinger (1987) also indicate that gravel-bounded streams with no lateral constraints such as cohesive materials or root reinforcement and are controlled only by hydraulic, excess shear stress conditions adjust to a Froude number near 1.0.

An associated theory of maximum sediment-transport efficiency (Kirkby, 1977) was also developed at this time using the corollary that the dissipation of minimum energy by the flow

signified maximum efficiency of bedload transport. If these parallel theories are to be substantiated with bedload-transport data, it holds that maximum, unit bedload transport should occur at a minimum specific energy, which equates to a Froude number of 1.0 (critical flow). The Froude number, representing the ratio between inertial to gravitational forces is an easily measurable metric obtained from discharge measurements.

Based on the parallel theories of minimization of energy and maximization of sediment-transport efficiency, the following hypothesis, can be stated and tested using bedload-transport and associated hydraulic data. That is "unit bedload-transport rates should peak at a Froude number close to 1.0, representing the maximum amount of bedload transport (per unit wdith) that can occur per unit of energy expended."

## SUMMARY OF FINDINGS

The hydraulic and sediment transport data used in this study come from about two decades of collecting and collating reports from the US Geological Survey and other agencies. Laboratory bedload-transport and hydraulic data were obtained from unpublished sources at the USDA-ARS, National Sedimentation Laboratory, and Colorado State University, as well as from the literature (ie. Gilbert, 1914; Stein, 1965; Foley, 1975; Nordin, 1976; and others). Field data were obtained from unpublished sources at the USGS Cascades Volcano Observatory, and from the literature. Sampling locations include the North Fork Toutle and Toutle Rivers, Washington (Dinehart, 1998); the Tanana River, Alaska (Burrows et al., 1979; Harrold and Burrows, 1983); the Jordan River, Israel (Inbar and Schick, 1979); and the East Fork River, Wyoming (Emmett, et al., 1982). The transport data cover sand- and gravel-sized bed material with median diameters ranging from 0.19 to 300 mm over Froude numbers ranging from 0.2 to greater than 3.0. All data were normalized by unit width to obtain unit-bedload transport rates ($q_s$) in kg/m/s and, by particle size (where $d_{50}$ of the material was available) to test the utility of predicting dimensionless unit bedload transport ($q^*$) across the range of sizes:

$$q^* = q_s \sqrt{\left(\frac{\rho_s - \rho}{\rho}\right) g d_{50}^{\ 3}} \qquad (2)$$

where $q^*$ = dimensionless bedload transport rate; $q_s$ = unit bedload transport rate, in kg/m/s; $\rho_s$ = sediment density, assuming 2.65 kg/m$^3$; $\rho$= density of water, assuming 1.0 kg/m$^3$; $d_{50}$ = median size of sediment, in m.

The various datasets indicate a reasonably tight relation (in semi-log space) between unit bedload transport rate and Froude number, with a peak transport rate occurring in the region of a Froude number of 1.0 (Figure 1). Transport rates increase sharply through the sub-critical regime, reach a flat peak at about 1.0 and then decrease slightly through the supercritical regime. This relation and its general form holds for the combined flume and field data over sand- and gravel-size ranges, and indicates that unit bedload-transport rates can be predicted with only measurements

of flow velocity and depth. Further study is required to determine the nature of the increased scatter in the vicinity of $F_r = 1.0$ and whether this is due to experimental and/or sampling issues.

Figure 1 Unit bedload-transport rate (per unit width) and Froude number

To check the consistency of the relation over the range of particle sizes represented by the data, the transport data are converted to the non-dimensional form by dividing by particle size. The resulting relation between $q^*$ (equation 2) and the Froude number is shown in Figure 2. Again we see a sharp increase in transport rates with increasing Froude number through the sub-critical flow regime, reaching a peak value in the region near $F_r = 1.0$. This plot predominantly includes the flume data. Particle size was missing from some of the extensive field datasets and will be incorporated in future analyses and publications.

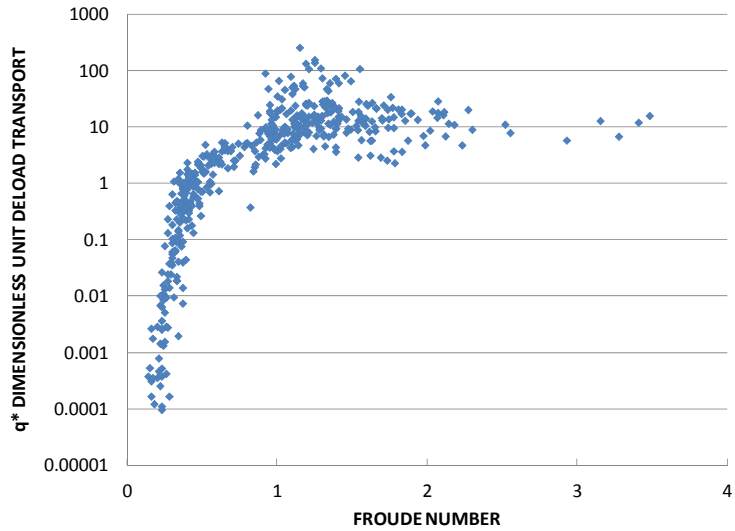


Figure 2 Variation in dimensionless unit bedload transport ($q^*$) with Froude number

The preliminary results from our analysis presented here support the hypothesis that the Froude number, i.e., a dimensionless relation between inertial and gravitational forces, has marked influence over unit bedload-transport rates, and that maximum rates occur at or near critical flow. This is an interesting development that could have great implications for sediment-transport theory and provide for a more simplified approach to estimating bedload-transport rates. Extension of this result from mechanistic sediment-transport theory to geomorphic-systems theory is also compelling. This work is still in progress. We are currently investigating the consistency of this relation across these and additional datasets after correcting for the effects of flume side walls and using dimensionless particle sizes rather than absolute values of grain size. We believe these steps will improve the resolution of this relation and that we can create a new, simplified sediment transport equation based on Froude number.

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