SAINT-VENANT MODELING FOR LARGE RIVER BASINS – CHALLENGES AND DATA NEEDS

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INTRODUCTION

River networks in continental-scale river basins – e.g. the $O(10^6)$ river miles in the Mississippi basin – to date have used only with reduced-physics models. These models cannot represent the momentum dynamics and water depth evolution available with the full Saint- Venant (SV) equations derived by cross-section integration of the Navier-Stokes equations. Momentum and water depth are valuable for mechanistic sediment modeling, and hence their neglect in reduced-physics models is a severe limitation. Reduced-physics models are generally justified with the argument that SV models are too computationally expensive for large networks (Hodges, 2013). However, recent advances in computational methods have made SV equations a practical option (Hodges and Liu, 2014; Liu and Hodges, 2014), but challenges remain in developing large-scale applications. In this presentation we discuss the problems, challenges, and some possible solutions for creating data sets combining surveyed and estimated channel geometry over large river basins. The key open question is with regard to data requirements: *what is the level of cross-sectional detail that is actually needed for SV solutions, vice the data detail that is typically obtained in surveys?* Ideas from hydraulic geometry combined with historical stage-discharge data can be used to develop estimated cross-sectional data, which are useful where surveyed data are unavailable. Although it is likely that for the foreseeable future we will not have comprehensive survey data for entire river basins, we can use available data to estimate geometry for dynamic equation solutions rather than a priori reducing the physical processes represented.

SPRNT MODEL

The Simulation Program for River Networks (SPRNT) was developed by Liu and Hodges (2014) to demonstrate that the SV equations can be solved efficiently for large river networks without linearization. This initial work showed that $O(10^5)$ computational elements in a river network could be solved for an unsteady SV solution $330\times$ faster than real time using an ordinary desktop workstation without any parallel processing. As is the case for any SV solution method, SPRNT input data includes: (i) the river network topology, (ii) the channel slope (S_0) of each element in the network, (iii) representative channel cross-sections for each element, and (iv) the channel roughness. SPRNT uses the conventional Manning's n and a SV formulation with flow rate (Q), and cross-sectional area (A) as solution variables:

$$\frac{\partial Q}{\partial t}+\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right)+gA\frac{\partial h}{\partial x}=gA\left(S_0-S_f\right) \tag{1}$$

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} + q_\ell = 0 \tag{2}$$

where S_f is the friction slope and q_ℓ is the lateral flux from the landscape. The local water depth (h) is an auxiliary function that depends on the cross-section geometry i.e. $h = h(A)$. Using the Chezy-Manning equation for S_f allows one to write $AS_f = n^2Q^2F$, where $F = P^{4/3}A^{-7/3}$ is an equivalent friction geometry with P as the wetted perimeter of the cross-section. Note that $F^{-1} = AR_h^{4/3}$, where R_h is the traditional hydraulic radius that appears in the Chezy-Manning equation:

$$Q = \frac{1}{n} AR_h^{2/3} S_f^{1/2} \tag{3}$$

Thus F is simply a convenient approach to wrapping the cross-sectional dimensional relationships of Chezy-Manning into a single term. In SPRNT the full nonlinear discrete equations are solved with an iterative Newton-Raphson method using acceleration methods that have previously been applied in microprocessor design.

DATA NEEDS FOR SAINT-VENANT MODELING

The critical data needed for SPRNT are not detailed x:y surveys of every cross-section, but instead for auxiliary functions for $h = h(A)$ and $F = F(A)$ that are abstractions of the cross section. Obviously, given discrete x:y survey data it is straightforward to compute discrete h:A and F:A for each element. However, in the absence of comprehensive surveys: how can we approximate the auxiliary functions from other data? As a further issue, the discrete h:A and F:A functions for a survey will generally have significant discontinuities such that naive application of raw data provides jacobians for the Newton-Raphson solution in SPRNT that are insufficiently smooth. That is, the Saint-Venant partial differential equations can only approximate a smooth system, which a priori requires $h(A)$ and $F(A)$ be smooth functions, which in turn requires $A(h)$ and $\partial A / dh$ must be smooth. Turning this idea on its head, if we have some approximation of $A(h)$ that is smooth, we can easily obtain a smooth wetted perimeter $P(h)$ that is consistent with $A(h)$, and hence compute $F(A)$ and $h(A)$ functions for a SV solution. Thus, the critical question for SV modeling is not the precise, detailed survey shape of the cross-section, but what is a smooth approximation of $A(h)$ that can be estimated from available data? We are investigating several avenues for this approximation, which will be discussed in the presentation.

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